Quantum Simulation of QCD within a particle-based encoding

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Abstract

Among the most anticipated applications of **quantum information science** is the **simulation of complex systems**. Those involving **quarks** and **gluons** are particularly compelling, as their **real-time phenomenology** remains elusive to traditional Monte Carlo methods. Overcoming these challenges could provide unprecedented insights into the **dynamics of partons.**

We discuss the calculation of **fragmentation functions**, key to describe how quarks and gluons transform into **observable hadrons**. As we move along we introduce a series of strategies to face the problem using quantum computers, all grounded in an encoding paradigm where **particles** and their internal degrees of freedom are the **central objects**.

Fragmentation functions

High-energy scattering:

1 - Protons are accelerated and collide

2 - The constituent partons (PDFs) interact **perturbatively**

3 - Final partons **recombine** with other particles to form hadrons (pions, protons, neutrons, kaons…)

$\Longrightarrow$ **Fragmentation functions:**

$$D_j^h(z) \equiv \frac{\mathrm{Tr}_c}{N_{c,j}} \sum_X \langle j,p|h,X_{\rm out}\rangle\langle h,X_{\rm out}|j,p\rangle$$

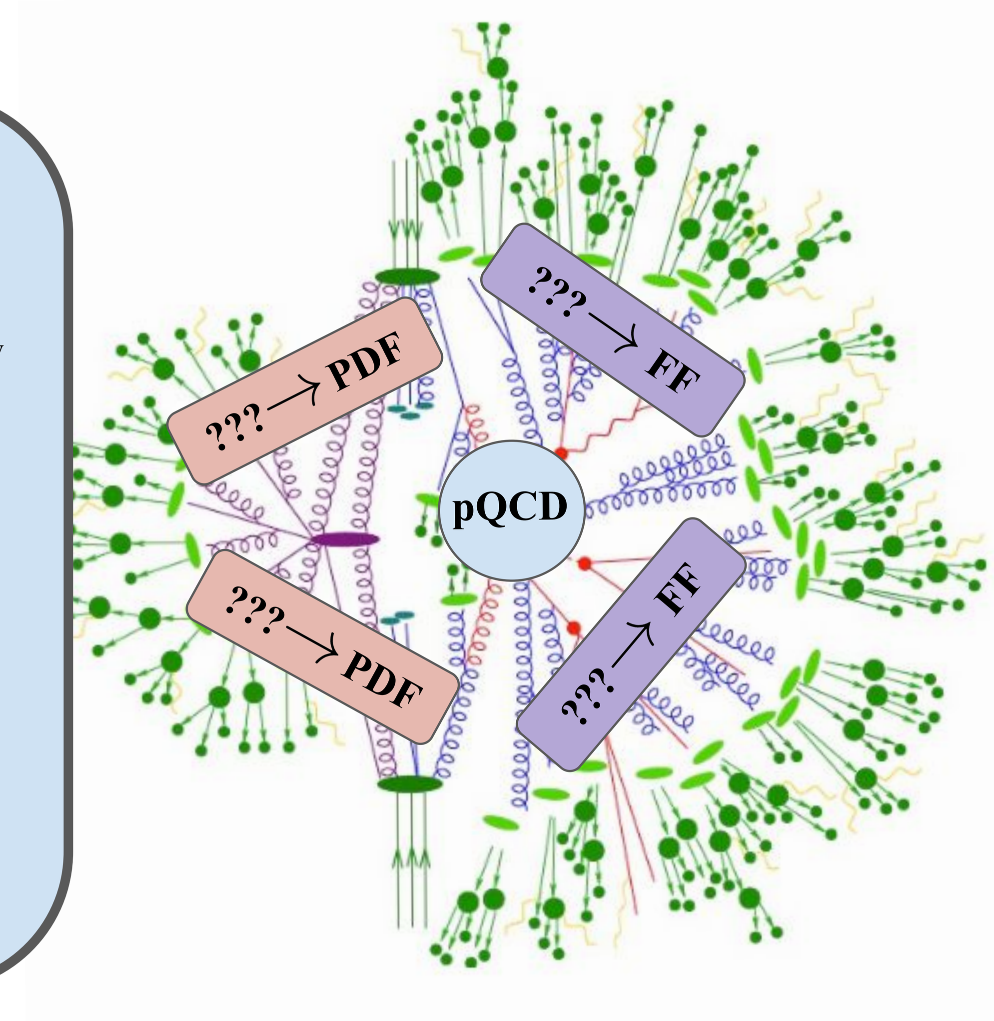
Probability to obtain **hadron *h*** from **quark *j***

z fraction of quark momentum carried by hadron

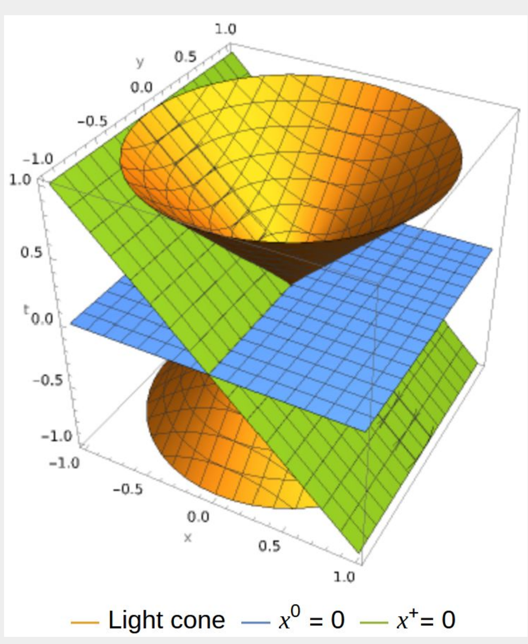
X are undetected states, summed over

Final hadrons far away in time and scattering direction, **evolve with Light-Front Hamiltonian:**

$$D_j^h(z) = \frac{\mathrm{Tr}_c}{N_{c,j}} \langle j,p|\, e^{-i\Delta x^+ P^-}\, a^\dagger_{h,p} a_{h,p}\, e^{i\Delta x^+ P^-}\, |j,p\rangle$$



QCD on the Light-Front



Coordinates

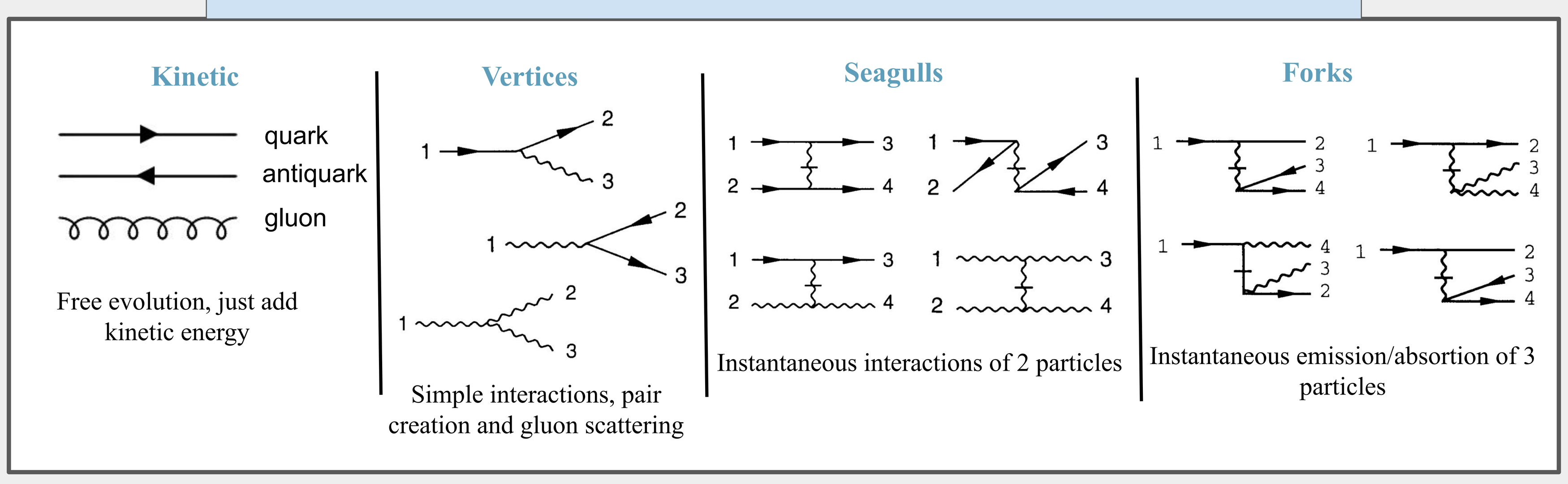
Instant Form (usual quantization)	Front Form
(x^0, x^1, x^2, x^3)	$x^+ = x^3 + x^0$, $x^- = x^3 - x^0$, $x^1, x^2 \longrightarrow x^\perp$
Energies: $p^0 = \sqrt{m^2 + \lvert p\rvert^2}$	$p^- = \frac{m^2 + \lvert p^\perp\rvert^2}{p^+}$
Gauges: $A^0 = 0$	$A^+ = 0$

From QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar\psi\,(i\gamma^\mu \partial_\mu - m)\,\psi + g\, A^a_\mu J^{\mu,a}_D$$

take $A^+ = 0$ and **Legendre transform** to obtain Hamiltonian $P^- = \sum \Pi\,\partial_+ A - \mathcal{L}$

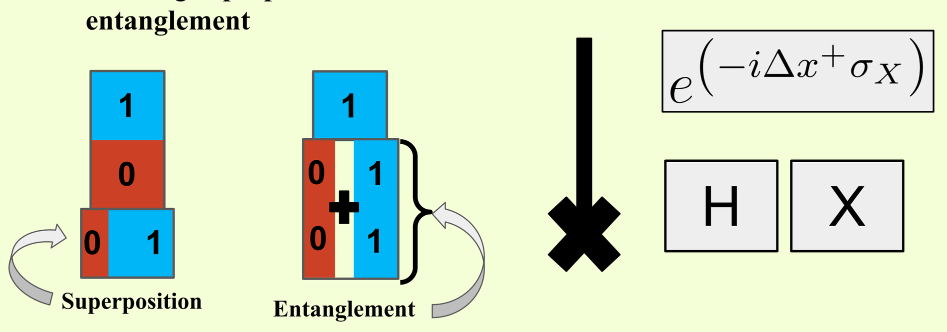
P^- can be divided into **interactions** that transform particle states



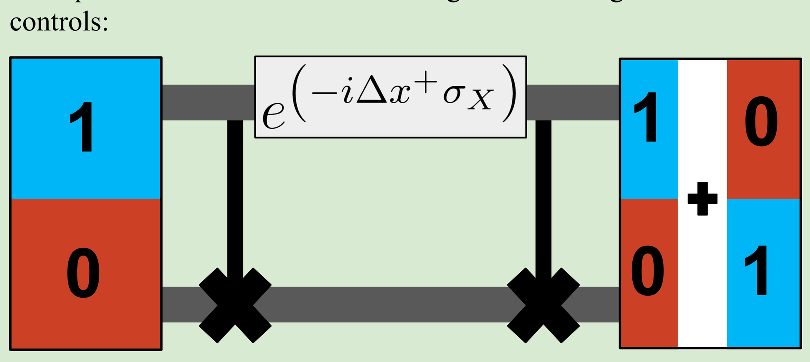
Quantum Computers & encoding

Quantum computers have two main components

- Quantum memories of qubits featuring **superpositions** and **entanglement**
- Logic gates to apply unitary evolution

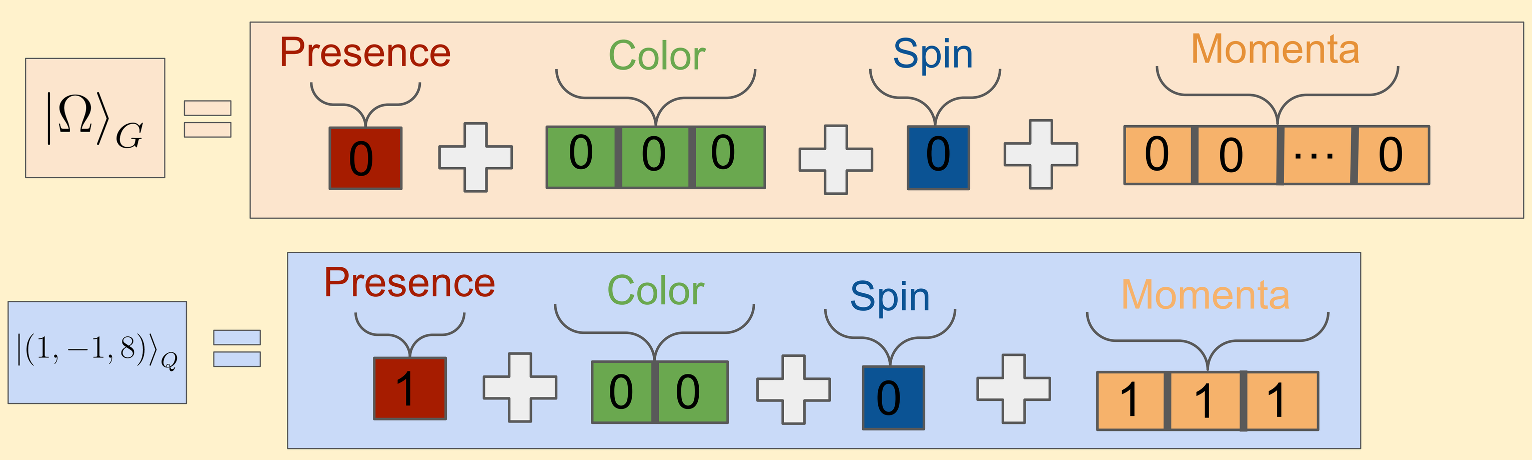


Two qubits can be evolved to an entangled state using rotations and controls:



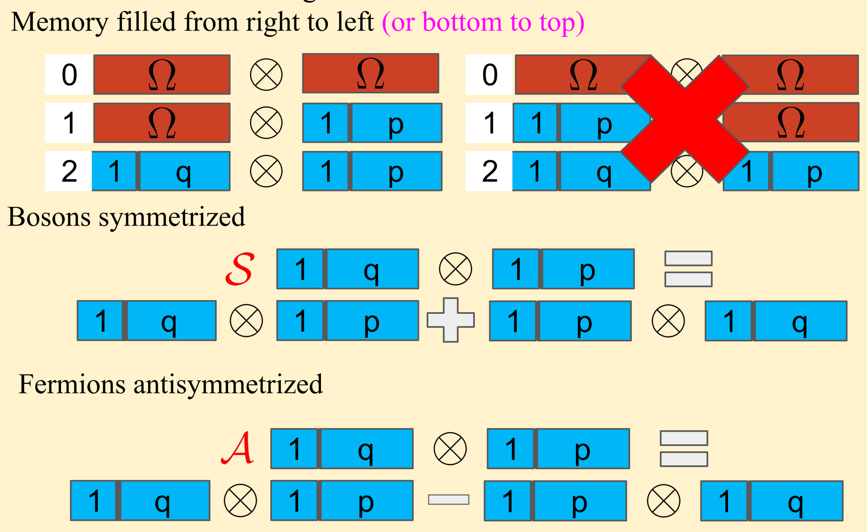
can we do the same with **particle states?**

Particle registers



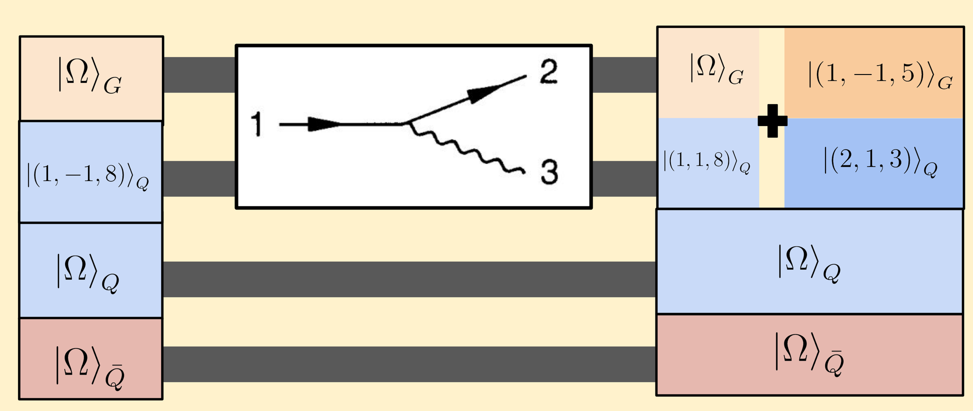
Conditions for correct encoding:

- Memory filled from right to left (or bottom to top)
- Bosons symmetrized
- Fermions antisymmetrized



Details in PhysRevD.110.116018

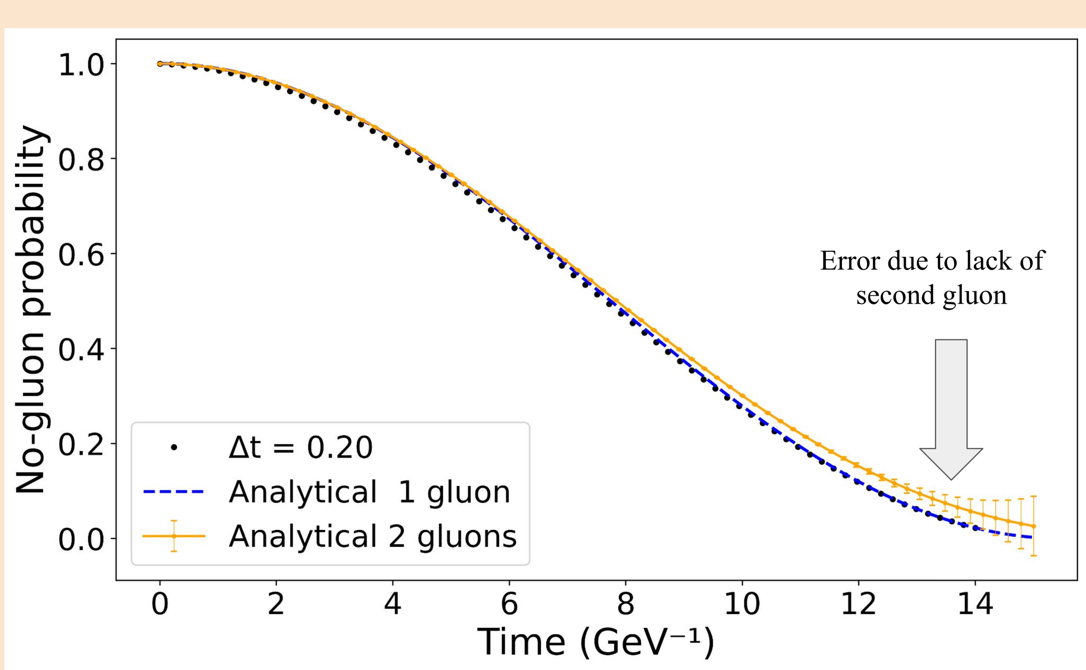
Add several together and apply rotations, particles become **entangled**:



Repeat for every term in the Hamiltonian to realize time evolution

Systematics due to Fock-space truncation

Classical simulators ⟶ very limited number of particles ⟶ errors due to Fock space truncation:



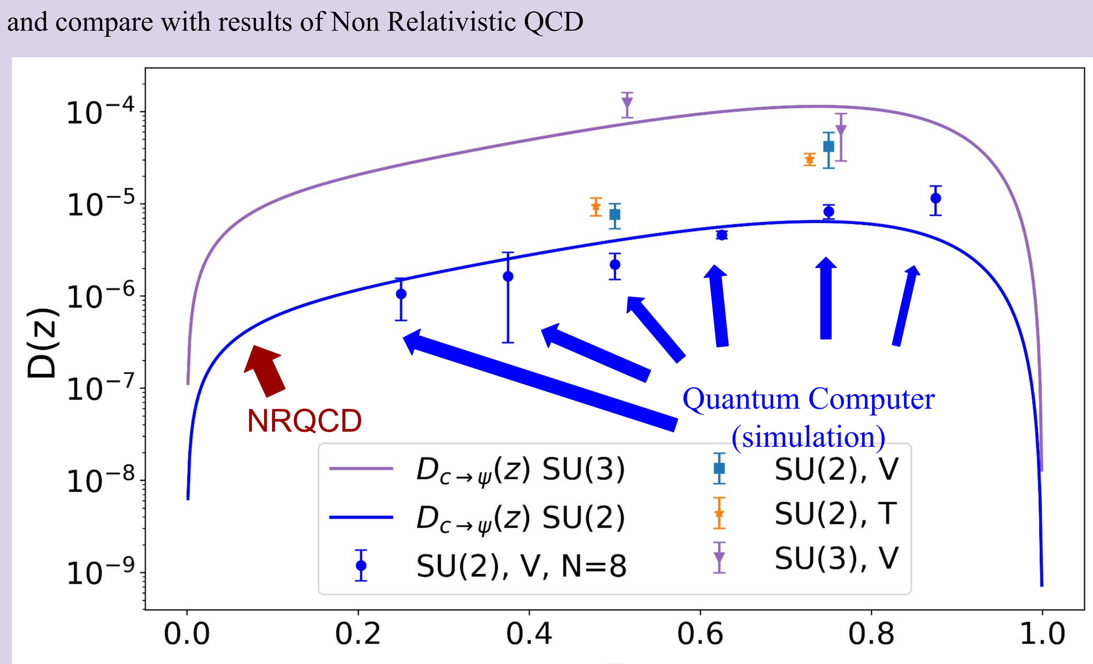
Fragmentation functions are measured before getting to the error zone

Example computation

Measure a simple meson of total momentum z after unitary evolution

$$|J/\Psi\rangle = \sum \frac{\delta_{c_q c_{\bar q}}}{\sqrt{x\,(z-x)}}\, \vec\sigma_{ij}\, \chi_0(x) |x\; i\, c_q, (z-x)\; j\, c_{\bar q}\rangle$$

and compare with results of Non Relativistic QCD



Conclusions

- Fragmentation functions with **2 quarks**, **1 antiquark**, and **1 gluon**.
- Total qubit count of 29 $\Longrightarrow$ few hundreds needed to solve the real problem
- Total number of gates $\sim 10^8$ far from today $\sim 10^3$ gates, but improving fast!
- In all: **Ab initio** calculation of fragmentation functions

Acknowledgements

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