Can ice sheets internally oscillate?

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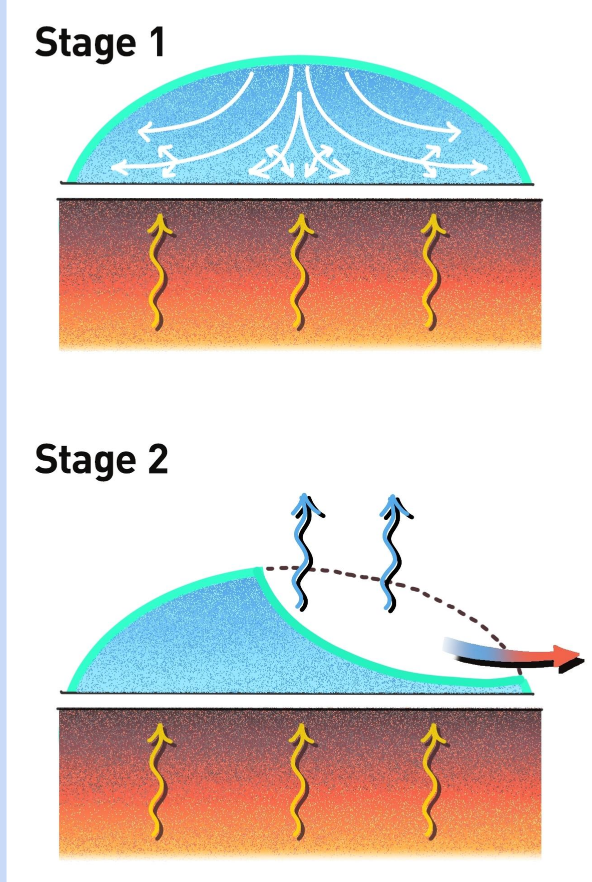
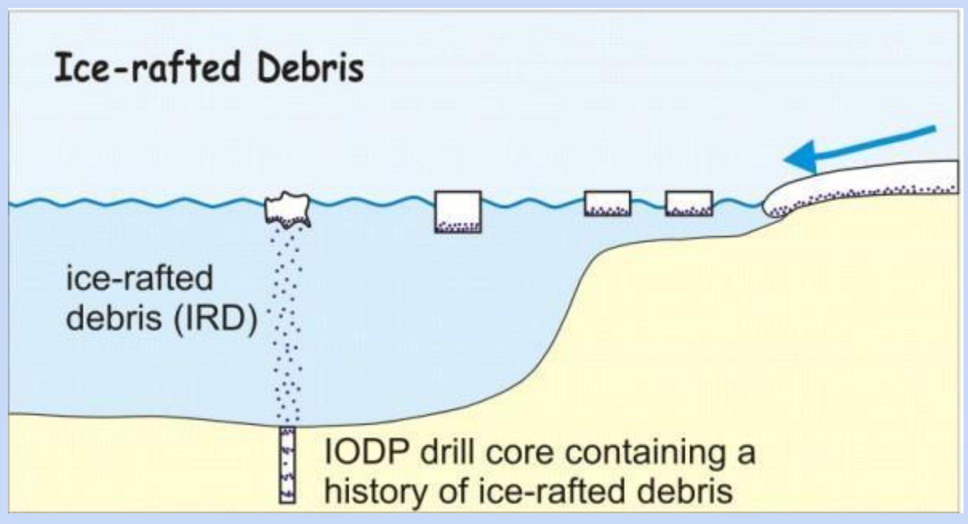
Introduction

The North Atlantic sediment cores contain **quasi-periodic layers** with extremely high percentages of lithic fragments during glacial periods. These sediments were captured by the ice, transported from the Northern Hemisphere ice sheets and eventually unloaded onto the seafloor when the enclosing ice melted. A non-homogenous sediment distribution suggests that ice flow might fluctuate in time. Several mechanisms have been proposed, among which internal-free oscillations rest on the assumption that there exists a transition between two potential states of basal lubrication.





Credits: Peter Hollinger.

Goals

- Estimate the periodicity with a more realistic geometry.
- Determine under what conditions internal oscillations are plausible.

Mechanism	Nature	Period	Caveats
Internal	Intrinsic to the ice sheet	$T \approx 7000$ years (MacAyeal, 1993)	Period estimation assumed an infinite domain

Proposed formulation: a finite 1D domain

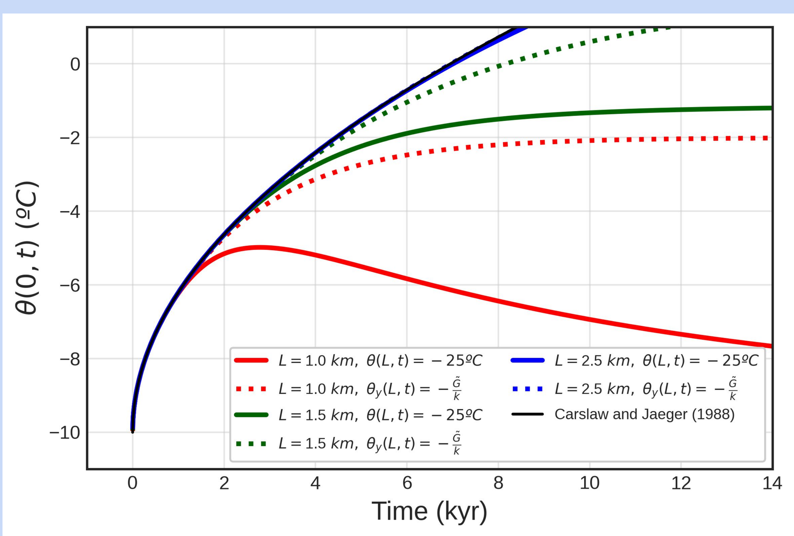
$$\begin{cases} \theta_t = \kappa \theta_{yy} + f & y \in \mathcal{I}, t > 0 \\ \theta(y,0) = \varphi(y) & y \in \mathcal{I} \\ \theta \text{ satisfies certain BCs.} \end{cases}$$

Separation of variables, "shifting data" technique ($\theta \rightarrow \zeta, \xi$) and series convergence lead to:

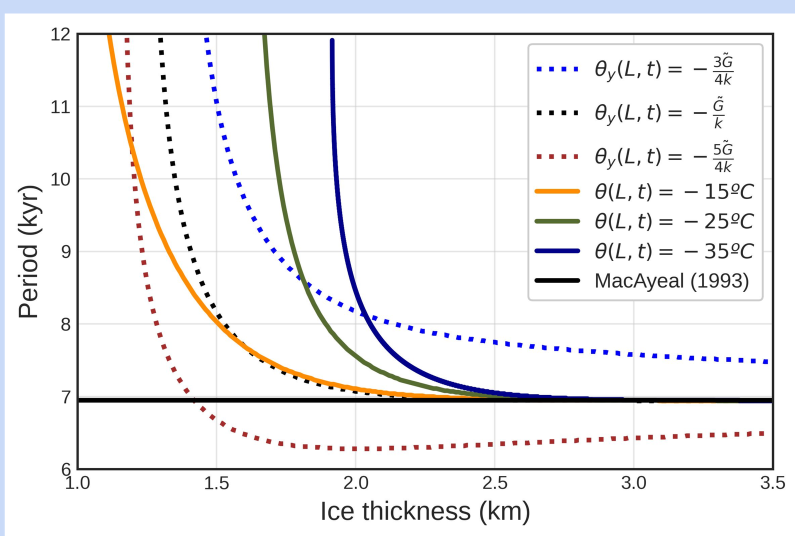
BC	Physics	Eigenvalues	Eigenfunctions
Robin	Fixed θ at the top	$\sqrt{\lambda_n} = \left(n + \frac{1}{2}\right)\frac{\pi}{L}$	$\zeta(y,t) = \sum_{n=0}^{\infty} A_n \cos\left(\sqrt{\lambda_n} y\right) e^{-\kappa \lambda_n t}$
Neumann	Constant ice-air θ_y	$\sqrt{\lambda_n} = \frac{n\pi}{L}$	$\xi(y,t) = \frac{\xi_0(t)}{2} + \sum_{n=1}^{\infty} \xi_n(t) \cos\left(\sqrt{\lambda_n} y\right)$

- Our solutions are strongly dependent on the ice thickness L.
- We retrieve $T \approx 7000$ years in the limit $L \rightarrow \infty$.

Temperature at the base $\theta(0,t)$.



Period of oscillation $T(L)$.

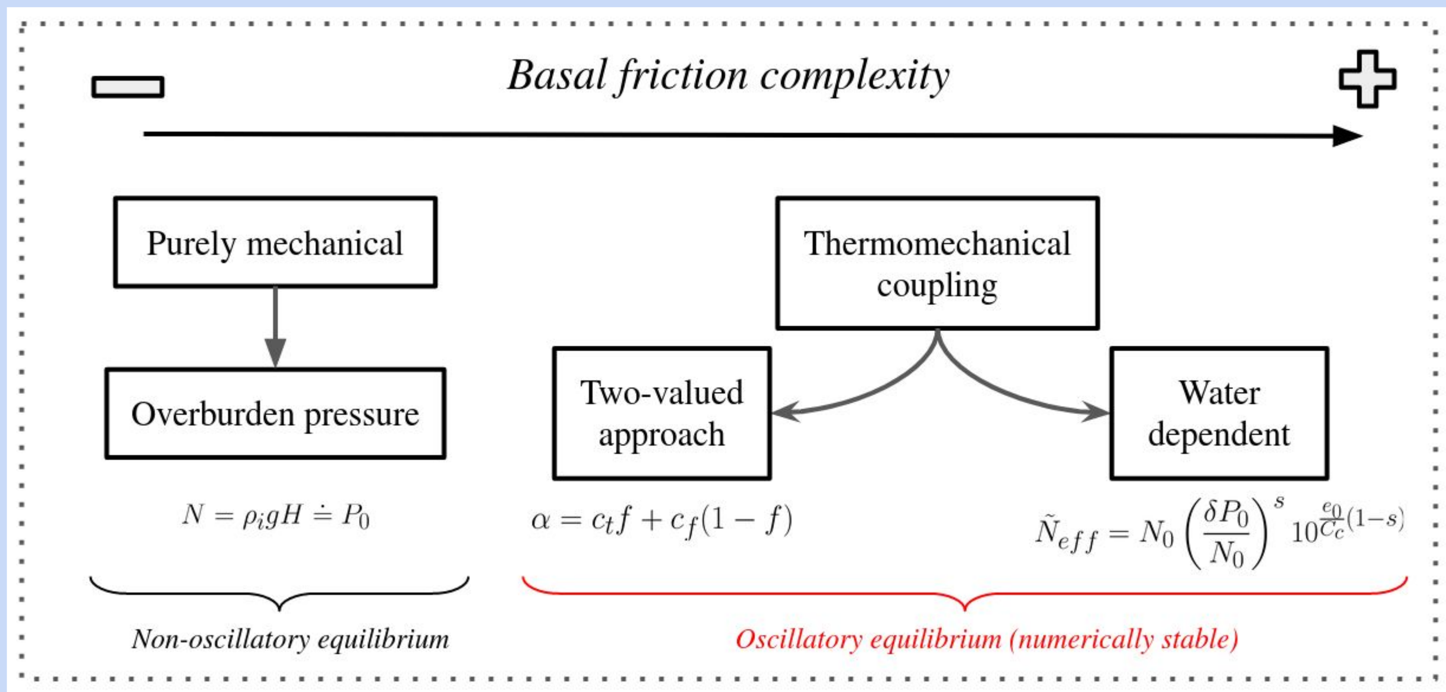


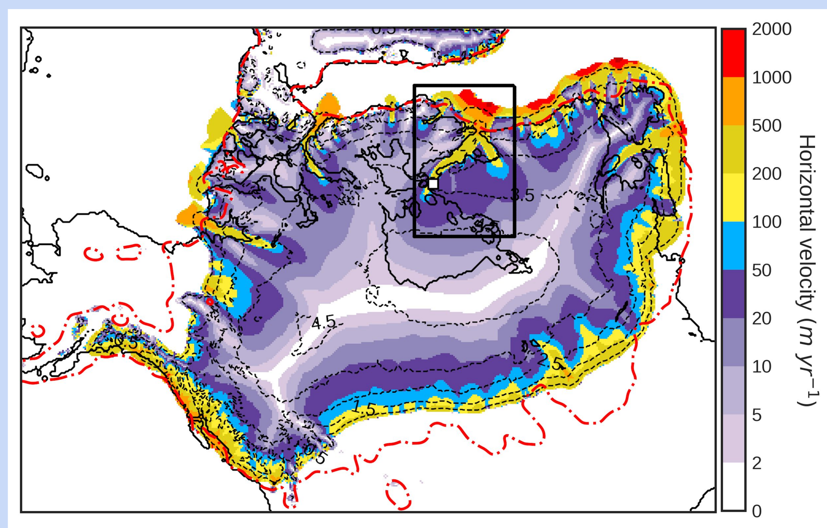
We further express Neumann solution in terms of Jacobi theta functions, thus eluding series truncation:

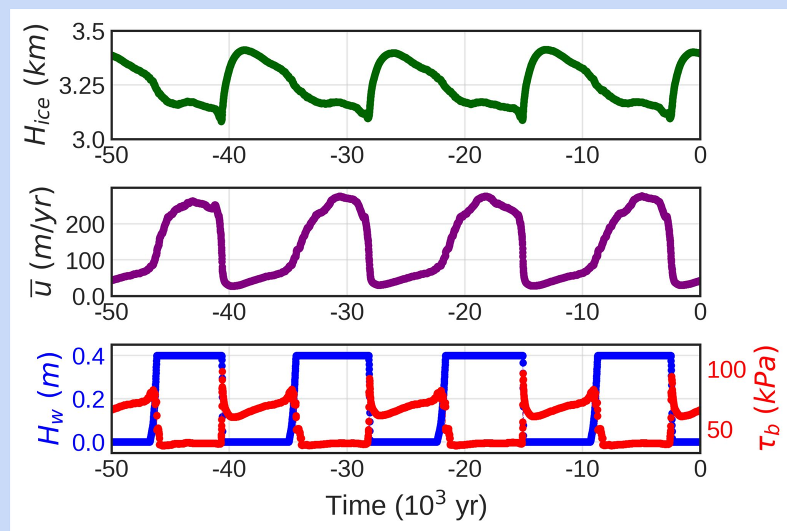
$$\xi(\tilde{y}, q) = -\frac{L}{2\pi^2} \left\{ \left(\theta_{y,air} - 3\frac{\tilde{G}}{k} \right) \log q - A\, \tilde{\vartheta}_3(\tilde{y}, q) + B\, \tilde{\vartheta}_4(\tilde{y}, q) \right\} + \xi(\tilde{y}, 1)$$

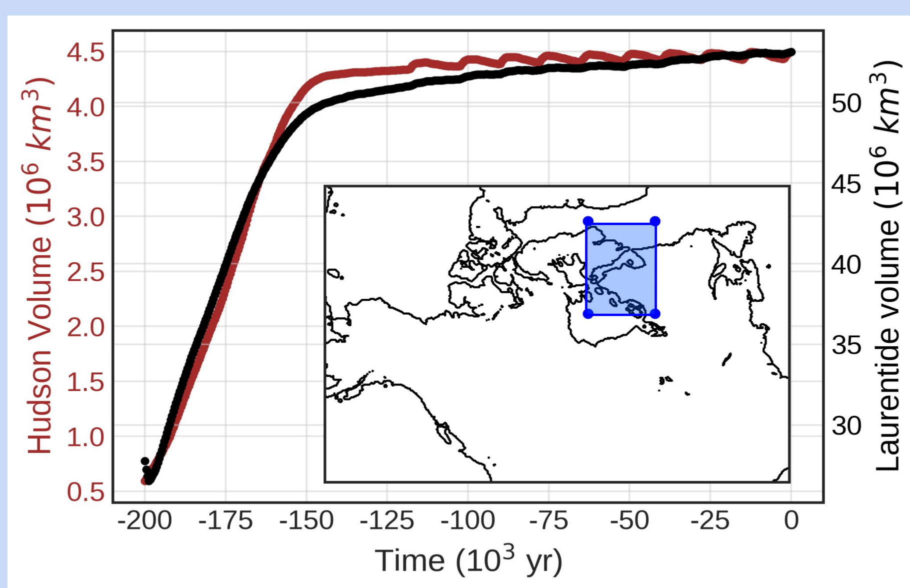
3D numerical simulations

- Higher-order thermomechanical ice-sheet model Yelmo (Robinson et al., 2020).
- Laurentide Ice Sheet (LIS) domain and constant BC.









1. Δx independent.
2. Volume nearly unchanged.

Conclusions

1. The assumption of an infinite domain in previous studies appears to be an over-simplification. Periodicity of internal-free oscillations is strongly dependent on ice thickness L, thus yielding two regimes.

2a. A thermomechanical coupling of the basal friction parametrization appears to be a necessary condition (yet not sufficient) for the existence of stationary oscillatory solutions.

2b. Volume changes seem unable to explain IRD layers.

References

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[3] Robinson, A., Alvarez-Solas, J., Montoya, M., Goelzer, H., Greve, R., and Ritz, C.: Description and validation of the ice-sheet model Yelmo (version 1.0), Geoscientific Model Development, 13, 2805–2823, https://doi.org/10.5194/gmd-13-2805-2020, 2020.