Studying the phase transition of the four dimensional Ising spin glass in magnetic field: a replica-symmetric Hamiltonian works

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1. Introduction

Glass formers (spin glasses, colloids, structural glasses, ...) display slow relaxations at low temperatures. The sluggish dynamics appears when large susceptibilities and correlations lengths are found. This behavior is usual in 2nd order phase transitions. Mean field theory predicts these features, but a crossover region could explain them too. So, it is unclear if a phase transition is present. Part of the difficulties arise from the complexity underlying field theory. Under some simplifying assumptions, Bray & Roberts proposed a replica symmetric field-theory [2]. Unfortunately, finding a stable fixed-point for the Renormalization Group flow has been problematic within the Bray & Roberts framework. In fact, no fixed-point has been identified for space dimensions D<6. Indeed, Höller & Read have suggested that the absence of a fixed-point is due to a 1st order transition [3]. Here, we study for the first dime several generalized susceptibilities suggested by the full field-theory [4]. We obtain the susceptibilities from D=4 spin-glass samples well equilibrated at low temperatures in the presence of a magnetic field (kindly provided by the Janus collaboration [5]). The behavior of these susceptibilities and renormalized coupling constants confirm the validity, as an effective field-theory at least, of the Bray & Roberts framework [1].

2. Theory and observables

We study the Ising spin glass model in a magnetic field h with N spins interacting via

$$\mathscr{H} = -\sum_{\langle x,y\rangle} J_{xy} S_x S_y + h\sum_x S_x, \qquad (1)$$

with the first sum is over nearest-neighbours pairs and $J_{xy} = \pm 1$ with 50% probability. We indicate thermal average as $\langle\cdots\rangle$, and the coupling average by an overline. The coupling average is performed after the thermal one, $\overline{\langle\cdots\rangle}$.

Using the replica formalism one obtain 3 linear susceptibilities, χ_j, and 8 coupling constant, $\mathscr{W}_i$. However, an expansion around mean field [4] finds that only the replicon susceptibility (χ_R) is divergent, while only two coupling constants, ω_1 and ω_2, are relevant near the transition. These relevant magnitudes are linear combination of the original ones.

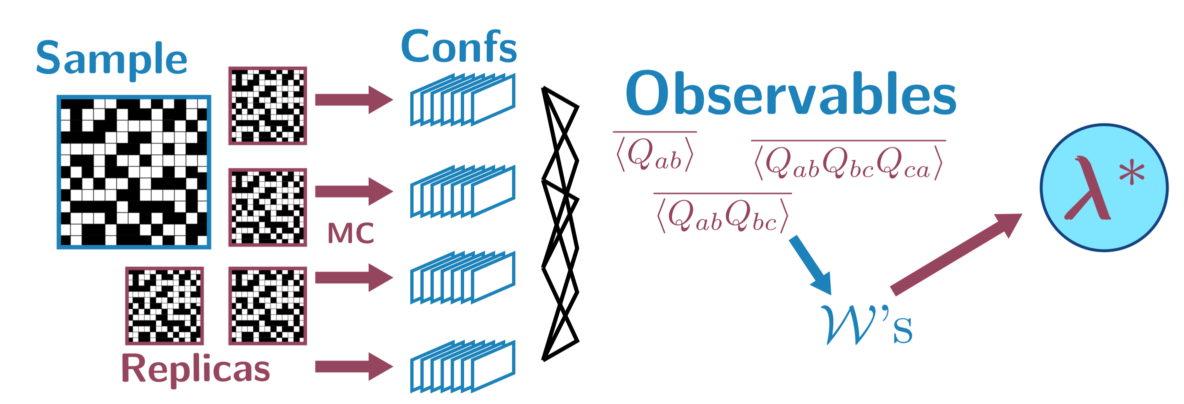
In principle, we need at least 6 different real replicas to estimate the original couplings constant $\mathscr{W}_i$. Nevertheless, the theory give us tools to estimate this couplings using only three replicas, $\omega_1^{(3R)}$ and $\omega_2^{(3R)}$, or four replicas, $\omega_1^{(4R)}$ and $\omega_2^{(4R)}$. Near the transition, these estimators approximate the true ω_1 and ω_2, and *coincide* with them at the critical temperature T_c.

All these magnitudes could be obtained from the replica overlap fields

$$Q_{ab} = \frac{1}{V}\sum_x S_x^a S_x^b,$$

and different products of them.

Finally, the parameter $\lambda \equiv \omega_2/\omega_1$ determine the nature of the transition. In particular, $\lambda > 1$ implies a 1st order transition, and $0 < \lambda < 1$ a 2nd order transition.

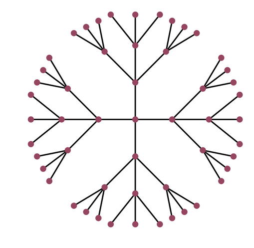


Graphical abstract of the process to estimate λ^*. We use the Monte Carlo configurations to calculate the thermal average, and from different samples we estimate the coupling average.

3. Results

Bethe lattice

The Bethe lattice spin glass offers a solvable setting to study the three- and four- replica estimators (in particular, the critical temperature T_c and $\lambda(T_c^+)$ are exactly known [6]). In Fig. 1 we plot the exact λ and both estimators. The estimators extrapolate to the correct value at the critical point. Notice the finite-size corrections, showing that $\lambda^* \neq \lambda(T_c^+)$.



Bethe lattice with 4 neighbors.

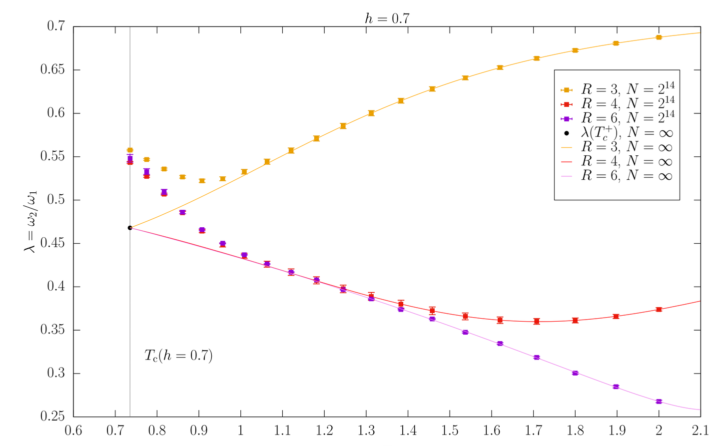


Figure 1: The three-, four- and six-replicas estimators of $\lambda(T)$, computed at magnetic field $h = 0.7$ on a Bethe lattice. T_c is marked with a vertical line. The black dot is the analytical value of $\lambda(T_c^+) \simeq 0.47$ [6]. The continuous lines are the extrapolations of the data considering scaling corrections.

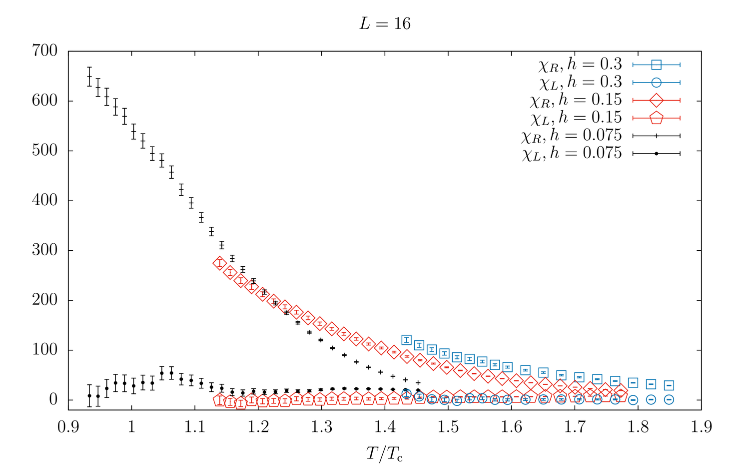


Figure 2: The value of the replicon susceptibility and of the longitudinal susceptibility for the largest systems simulated ($L = 16$) as function of the temperature in units of the estimated critical temperature for each magnetic field.

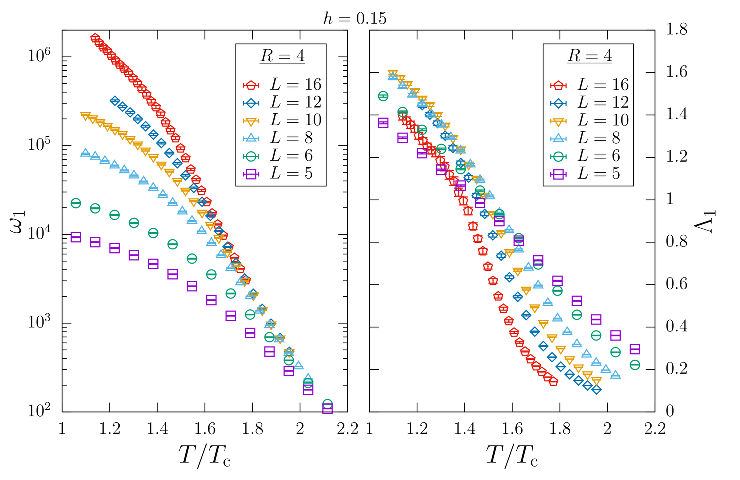


Figure 3: Left panel: ω_1 (4-replicas estimator) versus rescaled temperature in the 4D Ising spin glass with $h = 0.15$. Right panel: Λ_1 as a function of the rescaled temperature. Tc is obtained from [5].

Edward-Anderson 4D with field

A crucial check of the validity of the Bray & Roberts setting is mutual consistency of the three- and four-replica estimators. As a first step, we check that the susceptibilities behave as the theory predict. Indeed, see Fig. 2, the replicon susceptibility become extremely large, in contrast with the other susceptibilities. We have studied as well, the most diverging coupling constant ω_1 (Fig. 3 left panel). In fact, the quantity $\Lambda_1 \equiv \omega_1/(\chi_R^{3/2}\sqrt{V})$ (Fig. 3 right panel), should merge or cross for different system size L at the critical temperature.

Finally, in Fig. 4 we plot the ratio λ for the three- and four-replica estimators for different magnetic fields. For each magnetic field, both estimators for λ suggest an universal value $\lambda^* \sim 0.55$ at T_c. Furthermore, the two estimators approach their large-L limit from opposite sides. This helps in bracketing λ^*, verifying that $\lambda(L,T) < 1$. In addition, the small dependence on temperature for $T < T_c(h = 0.075)$ suggests that λ^* and $\lambda(T_c^+)$ are very close, so we conclude $\lambda(T_c^+) < 1$.

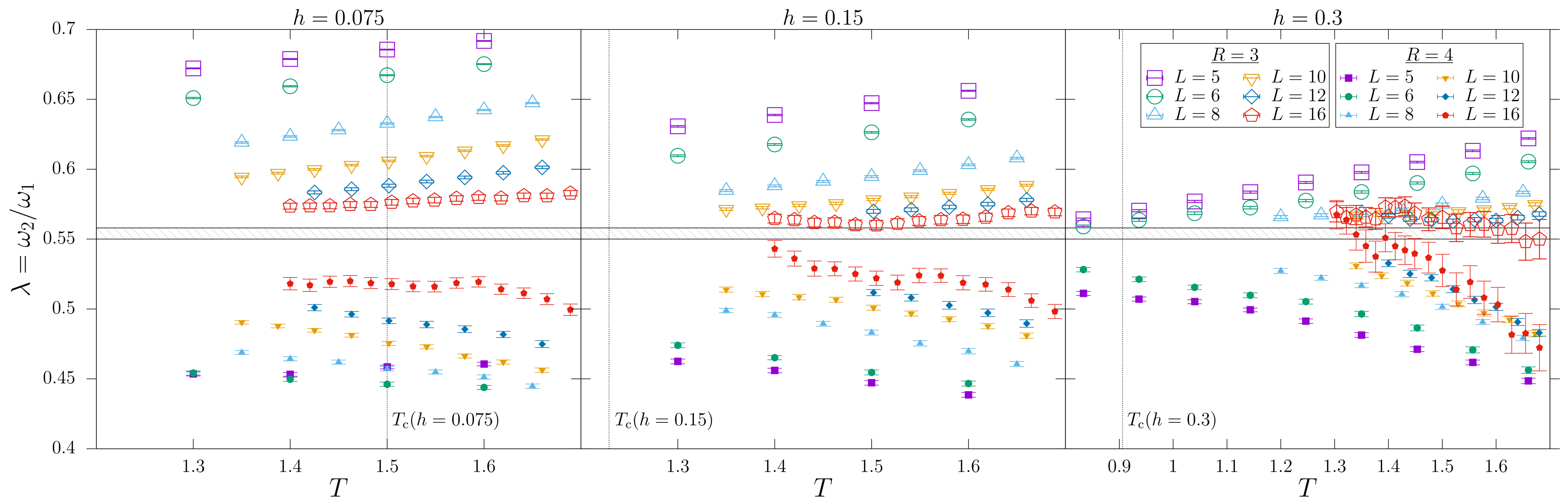


Figure 4: Three- and four-replicas estimators for λ as a function of the temperature in the four dimensional Ising spin glass (the value of the magnetic field h is indicated above each panel). Vertical lines report the three critical temperatures taken from [5]. The band around $\lambda^* \simeq 0.55$ is our best $L \to \infty$ extrapolation, assuming three- and four-replicas estimators converge to a common value for all the three simulated values of the magnetic field (the width of the band represents the uncertainty in our extrapolation for $h = 0.075$).

4. Conclusions

We have studied the generalized susceptibilities and renormalized couplings constants for the Ising spin glass in an external magnetic field, for both the 4D lattice and the Bethe lattice. The structure of the divergence is consistent with the predictions of the Replica Symmetric field theory. Up to our knowledge, this is the first validation of the assumptions underlying Bray & Roberts analysis.

References

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