A thermomechanically coupled flowline model

Daniel Moreno[1,2], **Marisa Montoya**[1,2], **Jorge Alvarez-Solas**[1,2] **and Alexander Robinson**[1,2,3]

[1]Department of Earth Science and Astrophysics, Universidad Complutense de Madrid, Madrid, Spain
[2]Geoscience Institute, CSIC-UCM, Madrid, Spain
[3]Postdam Institute for Climate Impact Research, Postdam, Germany

Flowline Github

Abstract

Ice-stream flow and grounding line migration remain as a main obstacle for glaciologists and manifests a clear a gap in our understanding of the physical phenomena underlying such behaviours. Here we present a thermomechanically-coupled flowline model to study a number of aspects regarding ice flow temporal variability and grounding line dynamics. Unlike previous studies, we consider the basal friction dependency on the sliding velocity and explicitly solve a coupled advection-diffusion Fourier heat equation. This allows us to quantify the dynamic and thermal contributions independently. Consequently, a solid understanding of the physical system will shed light on our comprehension of the relevant processes that determine the evolution of ice streaming and grounding line motion. We herein present a description of the numerical model and the successful results when tested against prior benchmark studies.

Introduction

Ice streams are a distinct feature of ice sheets with no counterpart in other geophysical thin-film flows. These regions of rapidly flowing ice exhibit velocities even three orders of magnitude faster than the usual glacial ice, yet they only account for a small fraction of the total ice sheet area (e.g., less than 5% of the Antarctic ice sheet).

The appropriate stress balance treatment is merely one of the challenges of ice streaming and the motion of an ice-sheet grounding line. Understanding the mechanisms governing its temporal variability remains as a major obstacle particularly at the aim of developing models of ice sheet dynamics (Robel et al., 2013). Given the broad range of ice flow speeds observed in real ice sheets, numerical simulations of these rapidly flowing bands are a well-known difficulty.

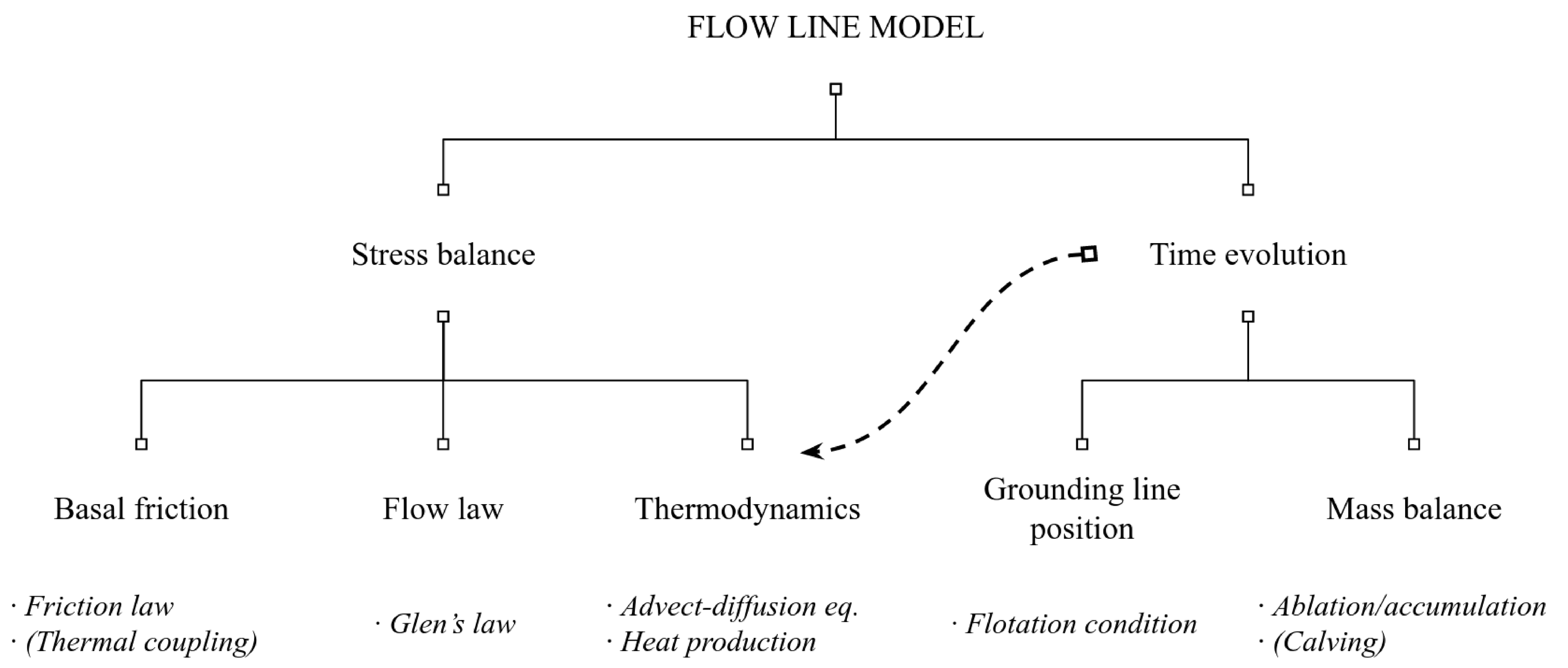

Figure 1: Conceptual view of the numerical model. The time evolution is determined by two hyperbolic equations, whereas the stress balance describes an equilibrium state of a simplified version (elliptic) of the Navier-Stokes equation.

We herein study the system formed by an idealised two-dimensional ice sheet geometry. Such system is described by a stress balance (Shallow Shelf Approximation, SSA) coupled to the advection equation (Eq. 1). Energy balance is explicitly accounted by a advection-diffusion heat equation. We numerically simulate such system by building a thermomechanically-coupled flow line model (Fig. 1).

Model description

We consider an ice slab of variable thickness $H(x)$ of two spatial dimensions: horizontal and vertical (x and z, respectively) by coupling the SSA stress balance and the advection equations. An appropriate formulation of the initial boundary value problem takes the form:

$$\begin{cases} H_t + (uH)_x = S(x), & x \in \mathcal{I},\ t > 0 \\ (4\eta H u_x)_x + \tau = \rho g H h_x & x \in \mathcal{I}. \\ H = H_0, & x \in \mathcal{I},\ t = 0. \\ h_x = 0, & x = x_0,\ t > 0 \\ u = 0, & x = x_0,\ t > 0 \\ 4\eta u_x = \rho g \frac{H^2}{2} - \rho_w g \frac{D^2}{2}, & x = L,\ t > 0. \end{cases} \tag{1}$$

where $u(x)$ is the ice velocity, ρ is the ice density, ρ_w is the water density, $\eta(x)$ represents the vertically-averaged ice viscosity and g is the gravitational acceleration. Subscripts denote partial differentiation.

Moreover, the ice temperature $\theta(x, z)$ is thought evolve in time through three main processes: vertical diffusion, horizontal advection and internal heat deformation of the ice.

$$\begin{cases} \rho c \theta_t = k\theta_{zz} - \rho c u \theta_x + \Phi, & x \in \mathcal{I},\ z \in \mathcal{L},\ t > 0, \\ \theta = \theta_0(z), & x \in \mathcal{I},\ z \in \mathcal{L},\ t = 0, \\ \theta_z = -G/k, & x \in \mathcal{I},\ z = \partial\mathcal{L}^-,\ t > 0, \\ \theta = \theta_L, & x \in \mathcal{I},\ z = \partial\mathcal{L}^+,\ t > 0, \end{cases} \tag{2}$$

where c is the ice heat capacity, G is the geothermal heat flux, k represents thermal conductivity, the internal heat deformation is denoted by Φ and θ_L is the surface boundary condition (constant temperature).

As a result, the ice viscosity η is thus dependent on both the strain rate. Lastly, basal friction τ is parametrised by three distinct formulations regarding its dependence upon the sliding velocity: a linear law, a pseudo-plastic power-law and a regularized-Coulomb.

We employ a stretched σ-coordinate system that allows us to explicitly track the position of the grounding line $L(t)$:

$$L_t = \frac{\frac{\rho_w}{\rho} D_t + (uH)_x - a}{H_x - \frac{\rho_w}{\rho} D_x}, \tag{3}$$

where $a(x)$ is the ice accumulation and D represents the sea level depth.

The model is written in C++ and further embedded in a Python environment. For further documentation, the code is available in an open Github repository: `https://github.com/d-morenop/flowline`.

Discretization

The system of Eqs. 1, 2 and 3 is discretized on a staggered grid (Fig. 2) and solved numerically by using finite differences (Fig. 2). The velocity field is determined by implictly solving the stress balance through a tridiagonal algorithm. Forward integration in time is then performed by an explicit scheme.

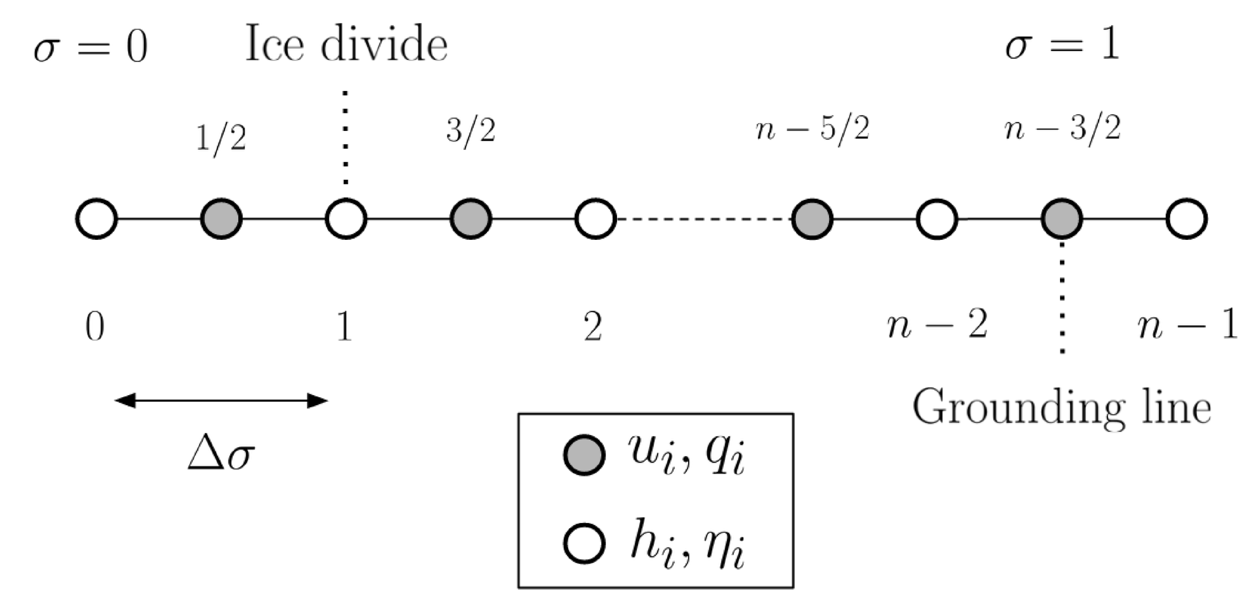

Figure 2: Schematic view of the domain discretization over a staggered grid. Vectorial quantities are evaluated over half-integers for numerical stability.

Results

We reproduce the Marine Ice Sheet Intercomparison Project (MISMIP, Pattyn et al., 2012) benchmark experiments as a test for our model. Results fall below a 5% deviation in ice thickness for the equilibrium solutions, exhibitting accuracy and robustness.

Schoof (2007) analytically proved that there are no stable solutions for a retrograde bedrock slope (i.e., positive slope values). Our model behaves accordingly and exhibit no equilibrium solutions in such a region (Fig. 3). Additionally, since the grounding line position can be analytically determined for a steady state, we can further compare our numerical solution (Fig. 4).

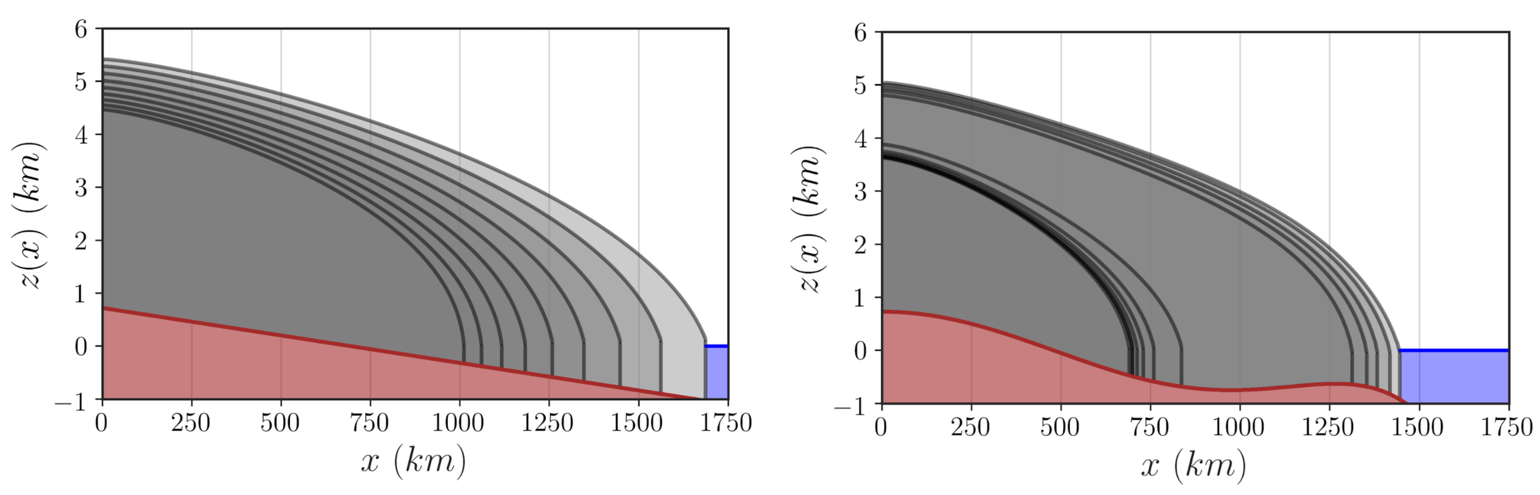

Figure 3: Equilibrium ice thickness profiles for a given set of constant A values (ice hardness) and two bedrock geometries: constant (left) and overdeepening (right).

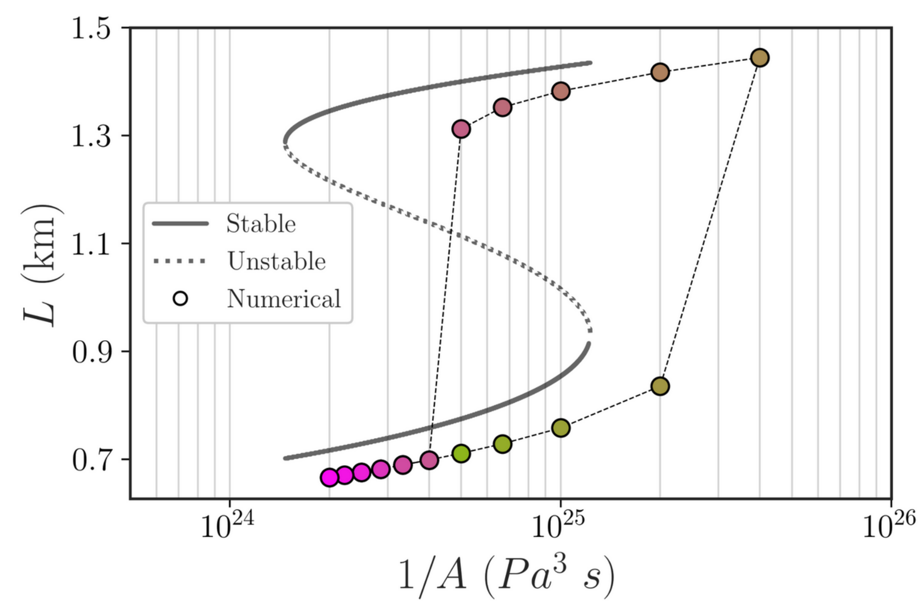

Figure 4: Comparison between numerical (coloured circles) and analytical (dark grey line) grounding line positions L at equilibrium for the hysteresis experiment. Total number of grid points $n = 250$.

Conclusions

- We have built an ice sheet model capable of reproducing all benchmark experiments of previous intercomparison projects (MISMIP).
- Our model behaves as predicted by the boundary layer theory (Schoof, 2007):
  - Equilibrium profiles are unique.
  - There are no steady solutions for a retrograde slope.
- Grounding line hysteresis is sensitive to the spatial resolution, yet we find convergence to the analytical solution.

References

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