Magnetic domains tune faster out-of-equilibrium critical dynamics [1]

I. González-Adalid Pemartín[1], E. Mompó[2], A. Lasanta[3,4], V. Martin-Mayor[1,5], and J. Salas[2,4]

[1]Departamento de Física Teórica, Universidad Complutense, Madrid, Spain
[2]Departamento de Matemáticas, Universidad Carlos III de Madrid, Leganés, Spain.
[3]Departamento de Álgebra, Facultad de Educación, Economía y Tecnología de Ceuta, Universidad de Granada, Ceuta, Spain.
[4]Grupo de Teorías de Campos y Física Estadística, Instituto Gregorio Millán, Universidad Carlos III de Madrid, Unidad Asociada al Instituto de Estructura de la Materia, CSIC, Spain
[5]Instituto de Biocomputación y Física de Sistemas Complejos (BIFI), Zaragoza, Spain

1. Introduction

Finding thermal equilibrium may be difficult, specially if one needs to do it fast. However, because reaching thermal equilibrium is a very important problem in industry, technology, and science, the non-equilibrium process are very studied, trying to answer the question:

How can we reach thermal equilibrium faster?

There are some examples of different non-equilibrium dynamics that allow a faster equilibration, like

- Mpemba effect [2],
- annealing techniques [3],
- stochastic reset.

In particular, Admit and Raz designed a strategy for system with time-scale separation [4]. Nevertheless, what happen with systems with a continuum time scale, e. g. a 2nd phase transition? So, in this work we try to answer the question:

What is the fastest way to reach thermal equilibrium in systems with a 2nd order phase transition?

Model

Ferromagnetic Ising model in 2D

$$\mathscr{H} = -J \sum_{\langle \mathbf{x},\mathbf{y}\rangle} \sigma_{\mathbf{x}}\sigma_{\mathbf{y}} \,, \qquad (1)$$

with $\sigma_{\mathbf{x}} = \{\pm 1\}$, $\langle\, ,\, \rangle$ the sum over nearest-neighbors,

in a thermal bath at inverse temperature $\kappa = \frac{J}{K_B T}$, energy units $J = 1$, and system size $L = 4096$. The system has a 2nd phase transition at $\kappa_c \simeq 0.44068$, which a paramagnetic phase (PM) for $\kappa < \kappa_c$, and a ferromagnetic phase (FM) for $\kappa_c < \kappa$. We know exactly $E_{\mathrm{eq}} = E(t \to \infty)$ [5]. We use the Metropolis (MET) and Heat-Bath (HB) dynamics rules (*model*-A universality class [6]).

Observables

Finally, in order to study our system we consider:

- the correlation function $C(\mathbf{r};t) = \frac{1}{L^2}\sum_{\mathbf{x}}\langle \sigma_{\mathbf{x}}(t)\sigma_{\mathbf{x}+\mathbf{r}}(t)\rangle$,
- the energy density $E(t) = -2C(\mathbf{r}_{\min};t)$,
- and the coherence length $\xi(t)$.

Thermal protocols

Isothermal Protocol (1P)

1. Generate a disordered spin configuration.
2. Put the configuration in a thermal bath at inverse-temperature κ $(t=0)$.
3. Evolve the configuration t Monte Carlo steps.

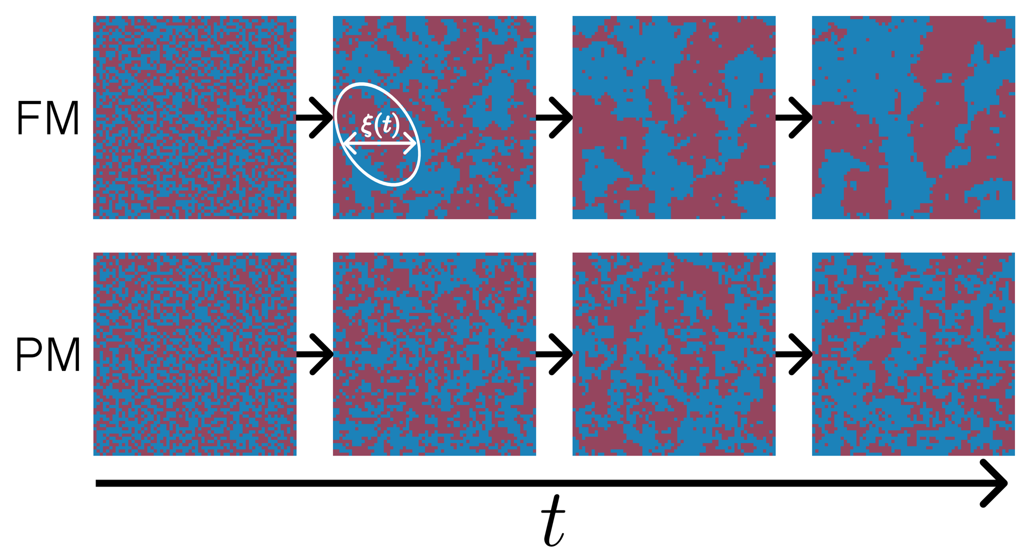

Figure 1: Cartoon of the system evolution using 1P. Schematic representation of the coherence length ξ.

Two-step Protocol (2P)

1. Generate a disordered spin configuration.
2. Put the configuration the FM phase.
3. Evolve the system until $\xi(t^*) = \xi_{\mathrm{start}}$.
4. At $t = 0$ heat back to the PM phase.

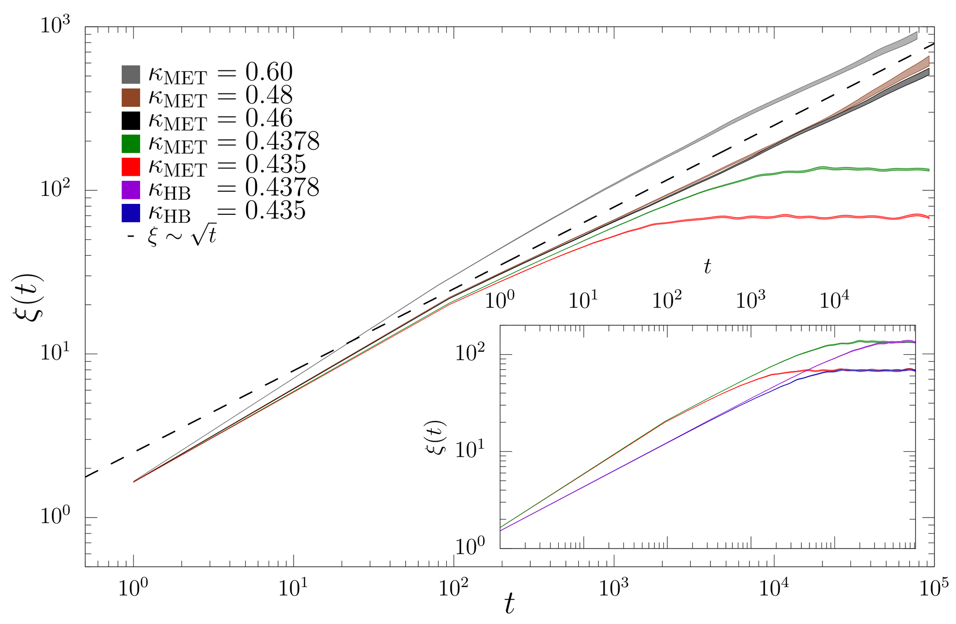

Figure 2: Coherence length ξ as a function of time t for 1P and MET (the width of the curves is twice the statistical error; κ increases bottom to top). **Inset:** Comparison of the HB and MET for $\kappa < \kappa_c$.

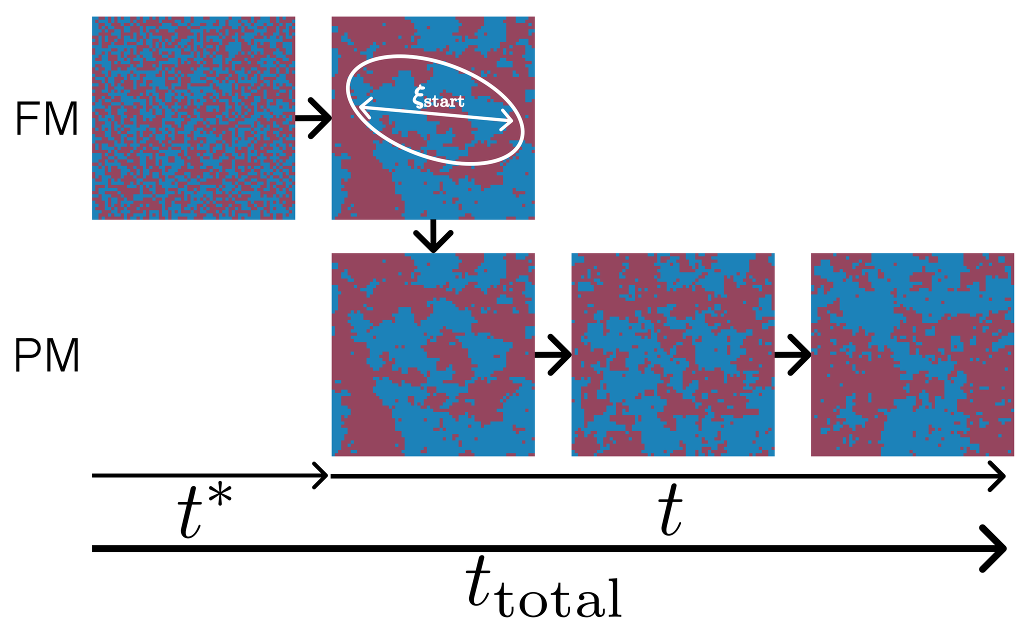

Figure 3: Cartoon of the system evolution in 2P.

2. Results

Isothermal evolution

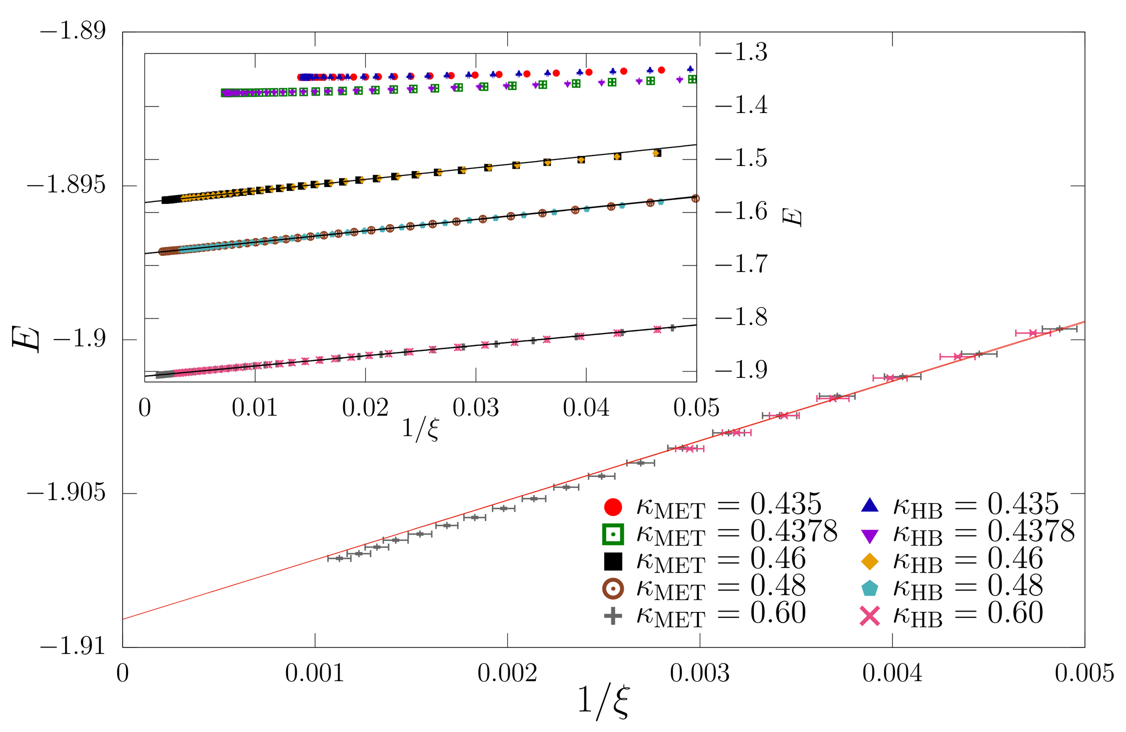

Figure 4: Energy density $E(t)$ from our 1P as a function of $1/\xi(t)$, computed for $\kappa = 0.6$. **Inset:** as main panel, for all our values of κ.

- In FM phase: $[E(t) - E_{\mathrm{eq}}] \propto 1/\xi(t)$. $\Rightarrow$ Extends to PM phase!

Can we use the FM faster magnet domains formation as an accelerator for the dynamics in the PM phase?

Exponential speed-up

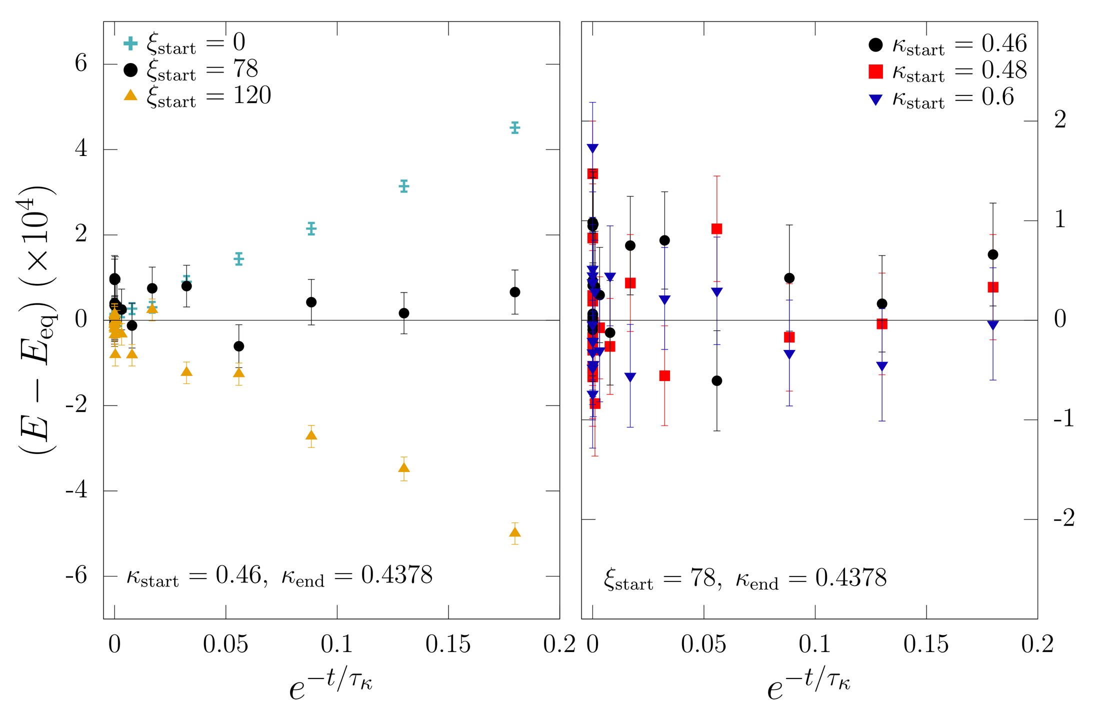

Figure 5: Excess energy $E(t) - E_{\mathrm{eq}}$ versus $\mathrm{e}^{-t/\tau_\kappa}$ (MET) for 2P (τ_κ is obtained by fitting our $\xi_{\mathrm{start}} = 0$ data).

- PM and $t \to \infty$: $E(t) - E_{\mathrm{eq}} = \mathscr{A}(\xi_{\mathrm{start}})\,\mathrm{e}^{-t/\tau_\kappa}$.
- $\mathscr{A}(\xi^*_{\mathrm{start}}) = 0 \Rightarrow$ Exponential speed-up.
- $\xi^*_{\mathrm{start}} \neq f(\kappa_{\mathrm{start}})$, and $(\kappa_c - \kappa_{\mathrm{end}})\xi(\kappa_{\mathrm{end}}) = cte$.
- $\lim_{\kappa \to \kappa_c^-} \left[\frac{\xi^*_{\mathrm{start}}}{\xi_{\mathrm{end}}}\right] = 0.59(7) \Rightarrow$ Universal for *model*-A.

3. Discussion

Real speed-up

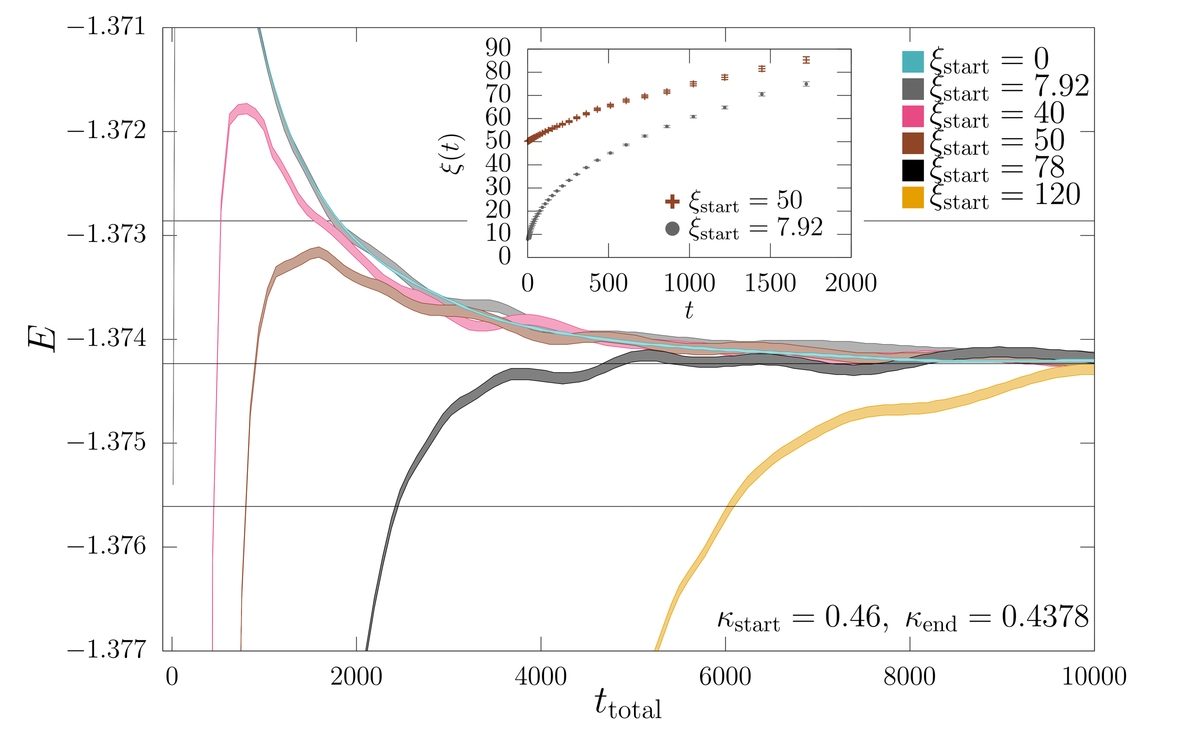

Figure 6: Energy density as a function of the total time elapsed since the beginning of 2P, t_{total} (MET). The width of the curves is twice the statistical errors (ξ_{start} increases from top to bottom). The horizontal lines correspond to E_{eq} multiplied by 1.001,1 and 0.999. **Inset:** coherence length during the 2P as a function of the time t.

Equilibration time

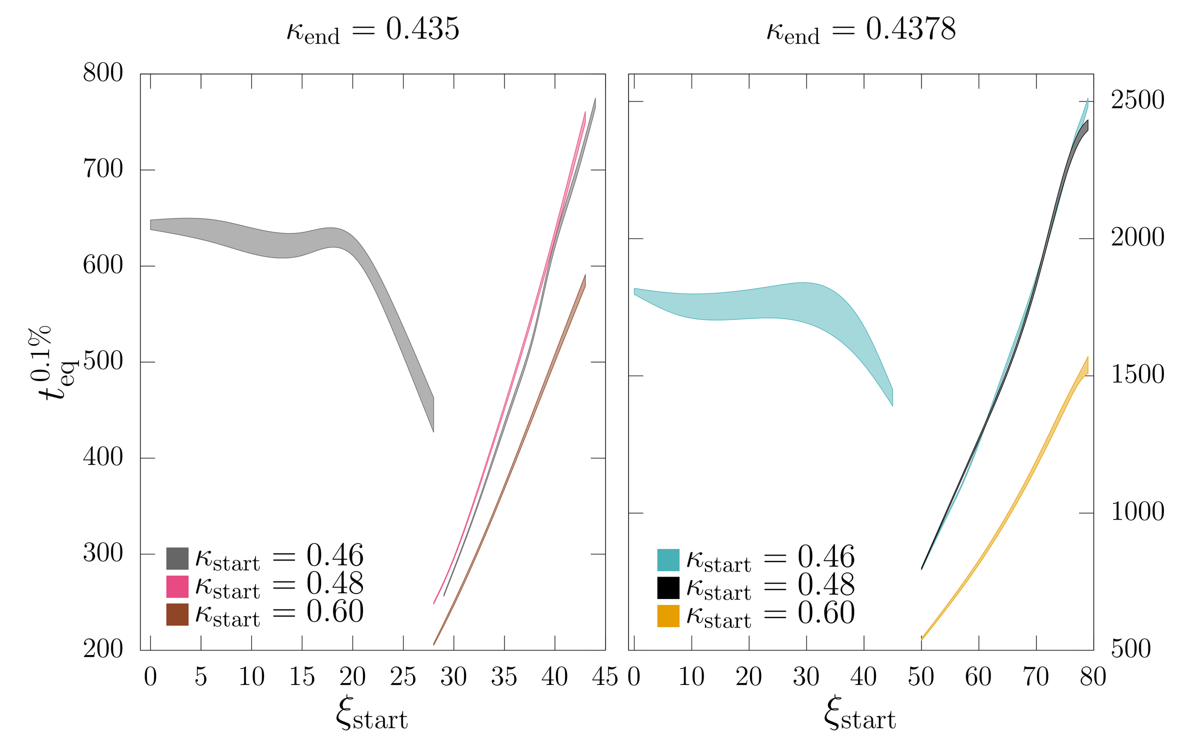

Figure 7: Equilibration time $t^{0.1\%}_{\mathrm{eq}}$ (defined as the last time the system has an energy with a difference less than a 0.1% from E_{eq}) versus ξ_{start} for 2P. versus ξ_{start} for 2P. The width of the curves is twice the statistical errors. The discontinuity is due to the non-monotonic time behavior of $E(t_{\mathrm{total}})$.

4. Conclusions

- Precooling may produce a faster equilibration at high T.
- The speed-up is driven by the magnetic domains in FM phase.
- An exponential speed-up is possible if the size of the magnetic domains is a well-defined fraction of the equilibrium correlation length in the high T phase.
- This will valid for the whole *model-A* universality class.
- We can use this mechanism with our *two-step* protocol.

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