Nonclassicality as an alternative resource for quantum metrology

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The sensitivity in the detection of weak signals is limited by the quantum mechanics according to the involved resources. *The minimum uncertainty achievable is usually linked to the energy of the system influenced by the signal.*

We show how this relation may be contradictory depending on the transformation applied. Alternatively, we study the dependence of the uncertainty on the nonclassicality implicated in the whole detection process.

Metrological scheme

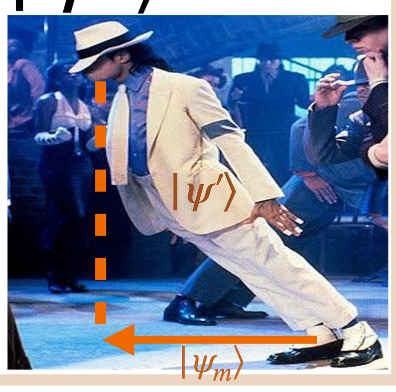
Probe state → **Signal encoding** → **Measurement**

$$|\psi\rangle \qquad e^{-i\lambda G}|\psi\rangle = |\psi'\rangle \qquad \langle\psi_m|\psi'\rangle$$

System which experiments the transformation

λ, *signal to be detected*
G, *generator of the transformation*

Measurement state, ψ_m



Limit of detection

Measurement statistics $P(\lambda)$

Fisher information $F(\lambda)$

Minimum uncertainty achievable:

$$\Delta^2\lambda \geq \frac{1}{F} \geq \frac{1}{4\Delta^2 G}$$

Cramer-Rao Lower Bound

The standard: ENERGY as resource

Probe state: *free particle.*

Energy: $\bar{H} = \frac{1}{2}\bar{P^2} = \frac{1}{2}\Delta^2 P = \frac{1}{2\Delta^2 X}$

Connection between the energy mean value of the system and the variance of two different operators.

Let's check the lower bound for each possible generator:

$\mathbf{G = P}$ $\quad \Delta^2\lambda \geq \frac{1}{4\Delta^2 P} \geq \frac{1}{\bar{H}}$

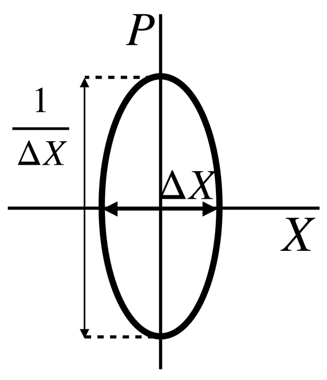
$\mathbf{G = X}$ $\quad \Delta^2\lambda \geq \frac{1}{4\Delta^2 X} = \frac{\bar{H}}{2}$

Contradictory dependence of the uncertainty on energy for both transformations!

~~Energy~~ is not the best option

Our approach: NONCLASSICALITY as resource

Probe state: *Squeezed coherent states*



$\Delta^2 X < 1 \Rightarrow$ **nonclassical state**

Nonclassicality $\propto 1 - \Delta^2 X$

Same test:

$\mathbf{G = P}$ $\quad \Delta^2\lambda \geq \frac{1}{F_P} \geq \Delta^2 X$

$\mathbf{G = X}$ $\quad \Delta^2\lambda \geq \frac{1}{F_X} \geq \Delta^2 P$

The more squeezing, the smaller uncertainty of the measurement

Equivalent relation between uncertainty and nonclassicality!

Nonclassicality is our resource!

• Contribution of the measurement

Nonclassicality of the measurement state is not negligible:

$$\frac{1}{F_P} = \Delta^2 X_0 + \Delta^2 X_m \qquad \frac{1}{F_X} = \Delta^2 P_0 + \Delta^2 P_m$$

• G = Number operator

$$F_N = \left(\Delta X_0^2 - \Delta Y_0^2\right)^2 F_X F_P + y_0^2 F_P + x_0^2 F_X$$

• G= X^2

$$F_{X^2} = \frac{\Delta X_0^2}{\Delta X_m^2}(F_X)^2 + 4x_0^2 F_X$$

- *Probe state: squeezed vacuum. Fluctuations may only be reduced until twice the minimum allowed.* ⟶ $F_{N_0} \leq 2\Delta^2 N = \frac{1}{2}[4\Delta^2 G]$

? Intermediate situation

- *Probe state: coherent state.* ⟶ $F_{N_\alpha} \leq \Delta^2 N = \frac{1}{4}[4\Delta^2 G]$

- Setting the amount of nonclassicality,
  $$\Delta^2 X_0 + \Delta^2 X_m = Cte,$$
  it is preferable to squeeze the probe state: $\Delta^2 X_0 \to 0, \quad \Delta^2 X_m \to 1$

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ACKNOWLEDGMENTS

L. A. and A. L. acknowledge financial support from Spanish Ministerio de Economía y Competitividad Project No. FIS2016-75199-P. L. A. acknowledges financial support from European Social Fund and the Spanish Ministerio de Ciencia Innovacón y Universidades, Contract Grant No. BES-2017-081942

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