Information can be recovered from thermal equilibrium

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ABSTRACT: We show that it is possible to recover information from the initial state of a quantum system after it has reached thermal equilibrium providing the process is done slowly enough. To do so, we perform a numerical simulation of a closed cycle bringing the system very slowly to a chaotic region where we let it reach thermal equilibrium and where the erasure of information would be expected. Next, we bring the system back to where it started very slowly. We obtain that the average expected value of macroscopic observables after the whole process depends on the initial conditions, therefore allowing the recovery of information about the initial state. We simulate the process using the Hamiltonian of the Dicke model through two different procedures, obtaining the same results.

1. Introduction

-A system is said to be chaotic if a small perturbation of the initial conditions leads to completely different trajectories.

-This has been proposed to be the foundation of ergodicity, which is the basic requirement that Classical Statistical Mechanics needs to calculate the time average values of any physical observable O without knowing the details of the movement of each one of its particles [2]:

$$(1) \qquad \langle O\rangle_t = \int_S O(\Gamma)\, d\Gamma \Big/ \int_S d\Gamma$$

-In Quantum Statistical Mechanics we have a radically different situation. An initial state, $|\Psi_o\rangle = \sum_E c_E |E\rangle$, evolves following the Schrödinger equation: $|\Psi(t)\rangle = \sum_E c_E e^{-iEt} |E\rangle$. This allows us to calculate the time-averaged expectation value of any physical observable Ô:

$$(2) \qquad \langle\langle \Psi(t)|\hat{O}|\Psi(t)\rangle\rangle_t = \sum_E |c_E|^2 \langle E|\hat{O}|E\rangle$$

-This depends on the initial condition. To solve this dependency the Eigenstate Thermalization Hypothesis establishes that Eq. (2) is equivalent to the microcanonical average at energy E:

$$(3) \qquad \langle \hat{O}\rangle_{micro,t} = \frac{1}{N}\sum_{E\in[E,E+\Delta E]} \langle E|\hat{O}|E\rangle$$

-This process of 'erasure' of initial conditions is called *thermalization*. Then, the previous discussion can be rephrased as the following statement:

Classic systems thermalize because they erase their initial condition. Quantum systems thermalize because their initial condition doesn't matter.

-Here we will show that there are some situations where this behavior leads to shocking conclusions.

2. Procedures

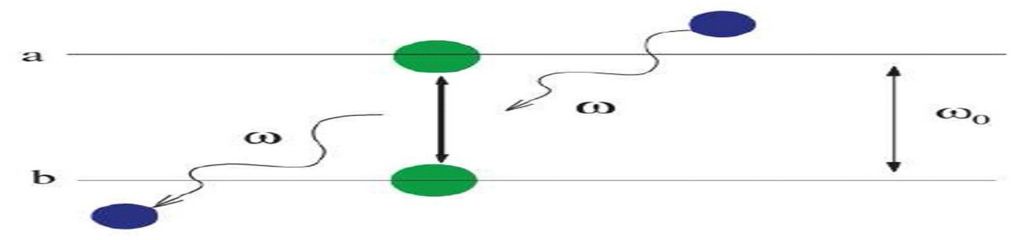
-We do two different simulations of a closed thermodynamic cycle: a succession of tiny *quenches* and a numerical resolution of the time-dependent Schrödinger equation.

-Both are done very slowly to guarantee adiabaticity and minimum energy dissipation.

-The model we simulate is described by the Dicke Hamiltonian,

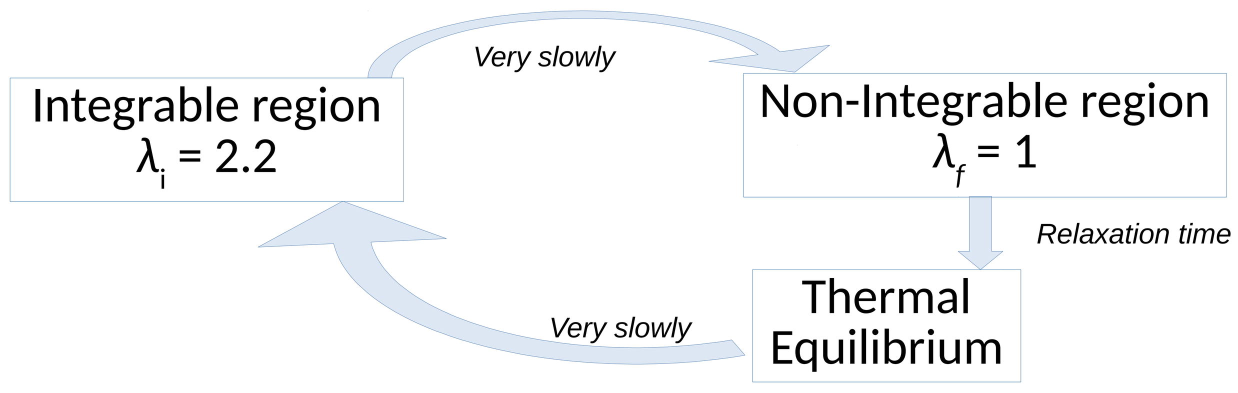
$$(4) \qquad H = w_o J_z + w\, a^\dagger a + \frac{2\lambda}{\sqrt{N}}(a^\dagger a + a) J_x$$

-The coupling parameter λ represents the strength of the interaction between the atoms (SU(2) algebra) and the photons ($a^\dagger a$), the constant ω is the frequency of the photons and ω_o is the energy gap between the two atoms:



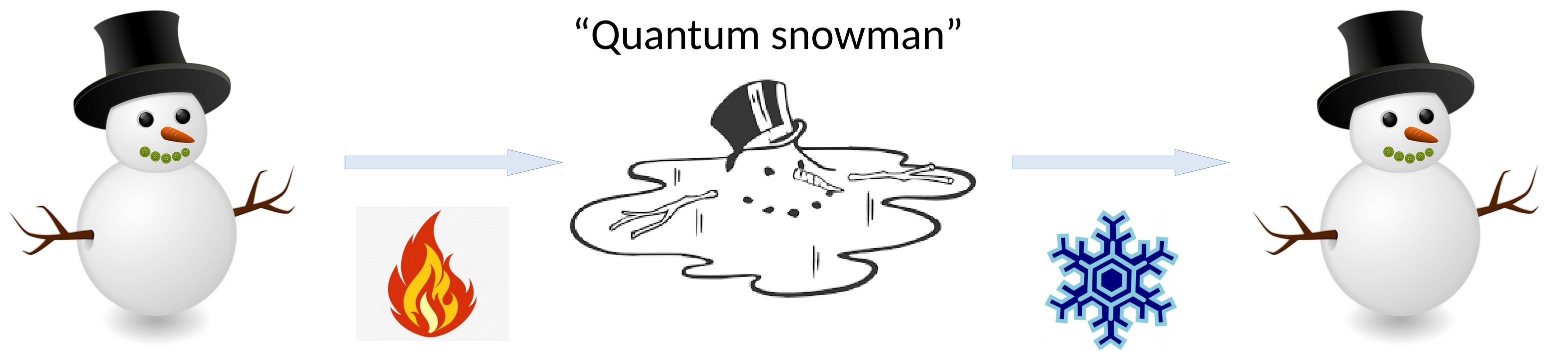
-This model shows chaos, excited-state phase transitions, band-structure [1], constants of motion [3], and avoided crossing of the energy levels and it is already being used in experimental research [4].

-We prepare a state in λ_i=2'2 in a way that holds some information (integrable region) and we do a closed cycle λ_i=2'2 → λ_f =1 → λ_i = 2'2:



Classical snowman

"Quantum snowman"



3. Results

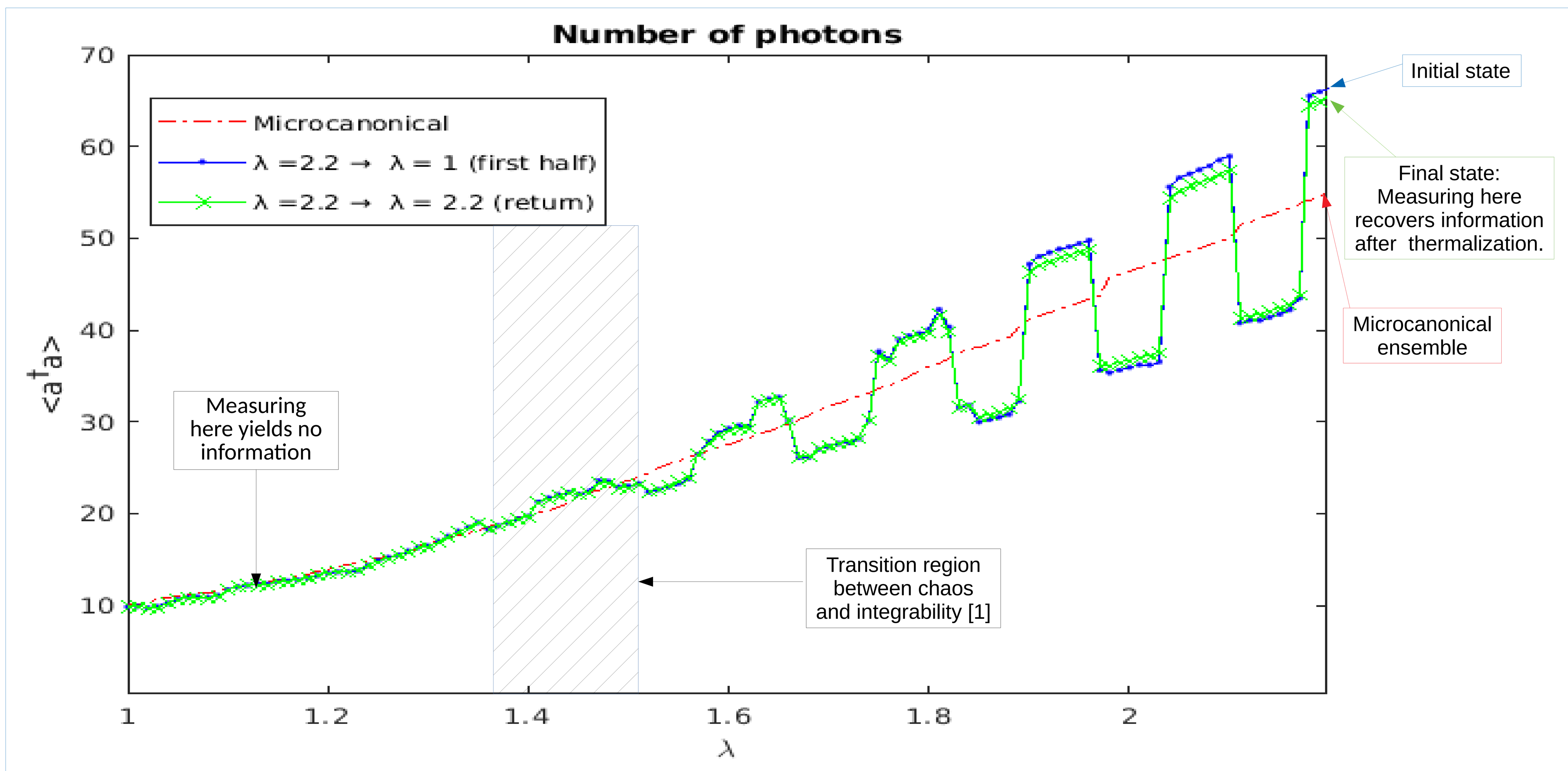


Figure 1: Number of photons at each stage of the cycle λ_i=2.2 → λ_f=1 → λ_i=2.2

-Red dashed line: microcanonical ensemble, using Eq. (3).

-Blue squared line: first half of the cycle (starting at the integrable region), using Eq. (2).

-Green exes line: second half of the cycle (return process) after letting the previous state (blue line) relax and reach thermal equilibrium, using Eq. (2).

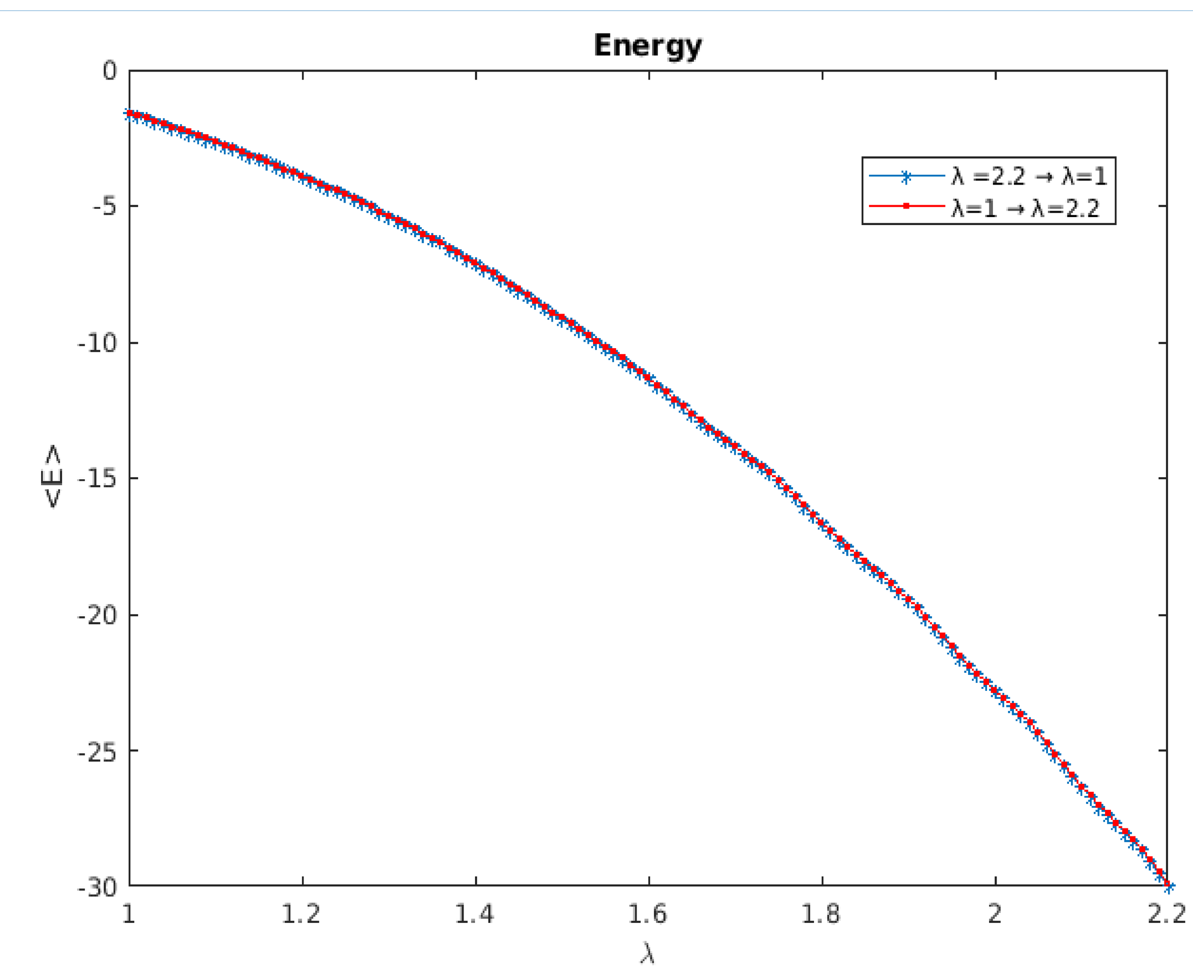


Figure 2: Energy is not dissipated at all during the process.

These graphs were obtained with the tiny-quenches procedure. Further research is yet being done with the other procedure (numerically solving the time-dependent Schrödinger equation), but so far we have obtained the same results.

We can clearly see that **the information stored in the system has been preserved after the whole process**, therefore allowing us to recover the original information of a system after it has reached thermal equilibrium.

4. Conclusions

-It is possible to recover the original information stored in a quantum system after it has reached thermal equilibrium.
-We suggest this mechanism to lead the system into an integrable region slowly enough and measure thermodynamics quantities.
-This model can be experimentally reproduced so these results can be tested.

5. References

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